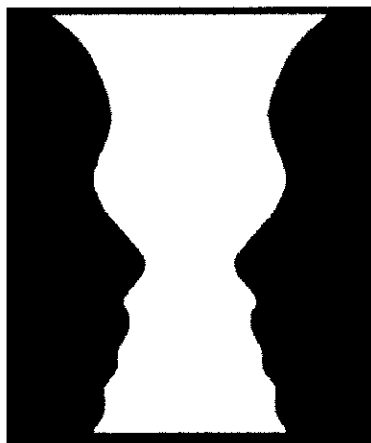# Life in an Insect World*

March 31, 2010

What do you see on the picture below? Write down your answer. Then look again and think if there is anything else there.

Each country in an insect world consists of several cities. Some cities are connected by tunnels. The insects are somewhat intelligent, but are poor at measuring distances and finding the most efficient routes. They know from their travels which cities are connected by roads and which are not. The insects are smart enough not to build two or more tunnels connecting the same pair of cities, or two build a tunnel connecting a city with itself. However, the insects do not have maps of their own countries.

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1. Here are the maps of the Antland and Beeland (see pictures below). Only you have these maps. The insects do not have them.


The insects of Antland talk over the phone with the insects of Beeland. They want to decide if their countries look the same. They start by asking

*How many cities do you have in your country?*

What questions can they ask next in order to decide whether their countries "look the same"?

4. QuadroLand has 4 cities. Draw all possible ways tunnels can join the cities in QuadroLand. (Remember that it's possible to have some cities not connected to each other).
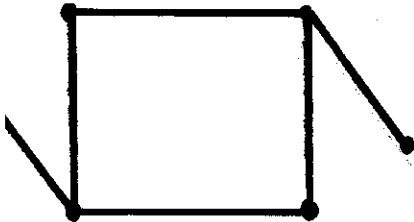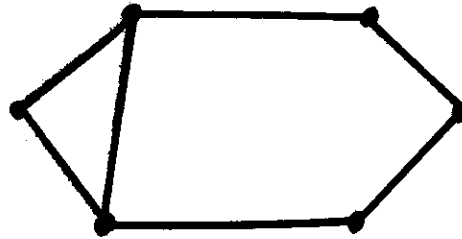
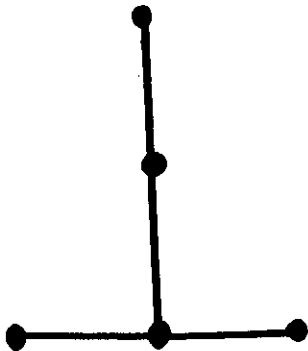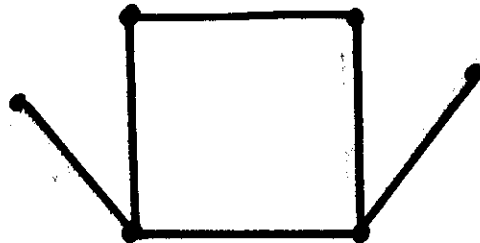2. Can the insects distinguish between the two countries below? Why or why not?
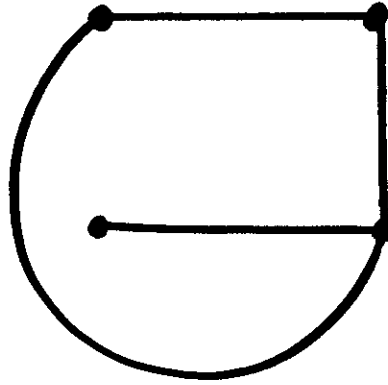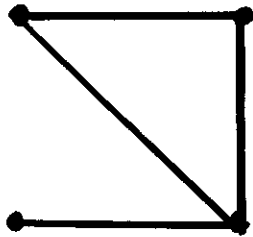
3. For each pair of countries below, decide whether insects would view them as the same or different. Notice that in all examples the number of cities and the number of tunnels are the same for both countries.
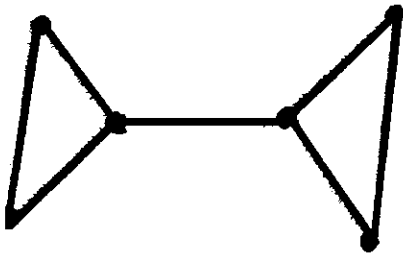
and

and
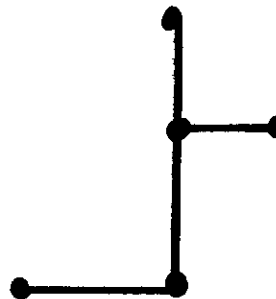

and

3

5. A wasp says that:

- there are 7 cities and 9 tunnels in its country;
- one city has just one tunnel connected to it;
- one city has five tunnels connected to it;
- two cities have three tunnels connected to them;
- the other three cities have two tunnels connected to them.

Draw a map that fits this description. Then draw another one! How can you tell these two countries apart?

6. Insects decided to decorate their cities by putting stars on their main towers in the following way. The number of stars they put on the main tower equals to the number of tunnels connected to this city.

(a) For the country below, write the number of the stars next to each of the cities;
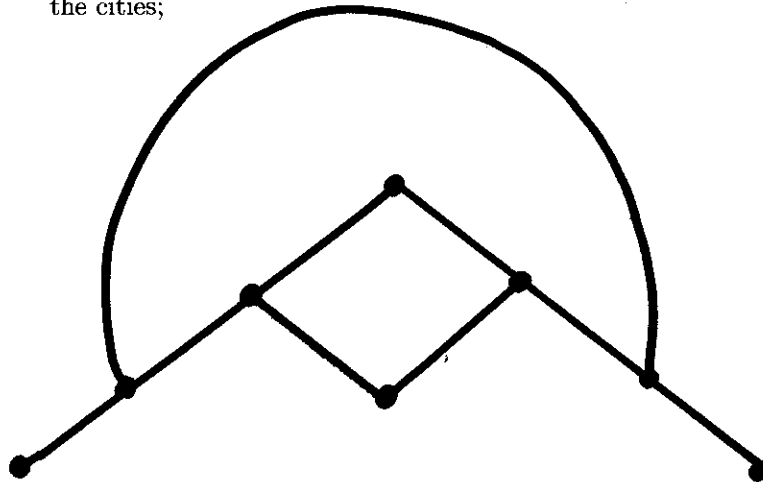
(b) What is the total number of stars this country will have (together in all cities)?

(c) How is the total number of stars and the number of tunnels in the country related?

(d) Can you explain this? Do you think this is always going to be the case?

7. Insects noticed that some of their cities have an even number of stars. They decided to call these cities "Even cities". All other cities are called "Odd Cities".

    (a) How many Odd Cities does the county below have?
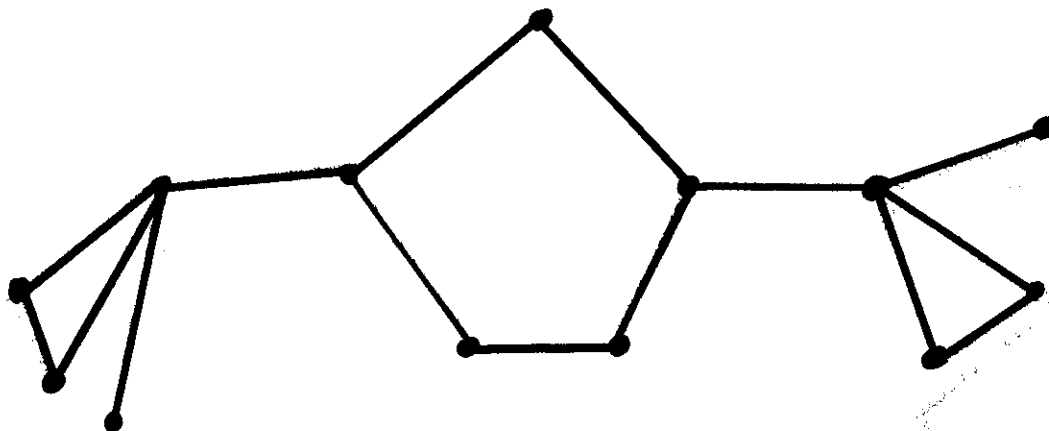
    (b) Draw your own insect country; Compute the number of stars for each city. How many Odd Cities does your country have? How many Even Cities does it have?

8. Insects noticed that in all of their countries

   The number of Odd Cities is even.

   Do you agree with them? If yes, they need your help proving this.

   (a) Insects started their proof by saying that in the beginning, a country does not have any tunnels. Thus, there are no stars on the city towers. What can you say about the number of Odd cities at this moment?

   (b) Suppose that at some moment insects build a new tunnel which connects two Odd Cities. How does this tunnel change the number of Odd cities?

   (c) Suppose that at some moment insects build a new tunnel which connects two Even Cities. How does this change the number of Odd cities?

   (d) Suppose that at some moment insects build a new tunnel which connects an Odd City with an Even city. How does this change the number of Odd cities?

   (e) Can you put together your observations in (a)-(e) to draw a conclusion about the number of Odd Cities?