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### Warm-up

**Problem 1** *When freezing water turns into ice, its volume expands by  $1/11$  of the original volume. What part of the volume does the ice lose when it melts back into water?*

**Problem 2** *A train line forms a closed loop (without self-intersections). 24 automatic trains circle the loop, keeping equal distances and moving at equal speeds. How many more trains moving at the same speed should be added to the loop so that the the distances between the trains shorten by  $1/5$ ?*

## Back to the book

**Problem 3** *Formulate the associative law of addition.*

**Problem 4** *Use the associative law of addition to find  $357 + 17999$  in an efficient manner. Use parenthesis to show all the steps of your solution!*

$$357 + 17999 =$$

**Problem 5** *Put the correct sign,  $>$ ,  $<$ , or  $=$ , between the fractions below. Do not use a calculator!*

$$\frac{12345}{54321}$$

$$\frac{12346}{54322}$$



- No fraction  $e/f$  s.t.  $e > 0$  and  $f < b + d$  exists in between  $a/b$  and  $c/d$ .

**Problem 7** How can you cut a  $1/2$ -meter-long piece from a  $2/3$ -meter-long string given nothing else but the string and scissors?

**Problem 8** Simplify the following expression as much as possible.

$$\frac{\frac{1}{a} - \frac{1}{b+c}}{\frac{1}{a} + \frac{1}{b+c}} \left( 1 + \frac{b^2 + c^2 - a^2}{2bc} \right) \div \frac{a - b - c}{abc}$$

Then find its value for  $a = 0.02$ ,  $b = -11.05$ , and  $c = 1.07$ .

**Problem 9** Simplify the following expression as much as possible.

$$\left( \frac{1+x+x^2}{2x+x^2} + 2 - \frac{1-x+x^2}{2x-x^2} \right)^{-1} (5-2x^2)$$

Then find its value for  $x = \sqrt{3.92}$ .

**Problem 10** *Red marks divide a stick into 7 equal segments. Green marks divide the same stick into 13 equal segments. The stick is cut into 20 equal pieces. Prove that, with the exception for the two end pieces, any piece has only one mark, red or green.*

**Problem 11** Given  $a + b = 11$  and  $ab = 21$ , find  $a^3 + b^3$ .

**Problem 12** Find the value of  $x^3 + 3x$  for  
 $x = \sqrt[3]{\sqrt{5} + 2} - \sqrt[3]{\sqrt{5} - 2}$ .



**Problem 13** *Solve the following equation.*

$$\frac{1}{x - \sqrt{x^2 - x}} - \frac{1}{x + \sqrt{x^2 - x}} = \sqrt{3}$$

**Problem 14** *Solve the following system.*

$$\begin{cases} \frac{1}{\sqrt{x}} + \frac{1}{\sqrt{y}} = \frac{4}{3} \\ xy = 9 \end{cases}$$

**Problem 15** *Solve the equation  $3x^3 + 2\sqrt{3}x^2 - 21x + 6\sqrt{3} = 0$  given that the product of two of its roots equals 1.*

**Problem 16** *Given that the roots of the equation  $x^3 + ax^2 + bx + c = 0$  form a geometric sequence, prove that one of them equals  $-\sqrt[3]{c}$ .*

## Powers

**Problem 17** Put the correct sign,  $>$ ,  $<$ , or  $=$ , between the following numbers.

$$10^{-3} \quad 2^{-10}$$

**Problem 18** Given  $2^{1000}/2^n = 2^{510}$ , find  $n$ .

**Problem 19** How many decimal digits do you need to write  $2^{100}$ ?

**Problem 20** *A city population grows 5% every year. How long would it take to triple?*