

## Junior Circle Meeting #9 – Logic and Mirror Problems

March 7, 2010

### Logic

An *implication* is a two-part statement where one statement implies the next. The form of an implication is usually, “If ..., then...”

For example, “If I come to Math Circle today, then I will learn something new”, is an implication.

An implication can either be true or false. Let’s examine this now:

1. Mira said:

“If I go to the movies, then I will buy popcorn.”

She did not go to the movies, but she bought some popcorn from the store. Can someone say that Mira lied? Why or why not?

Let’s say A= “going to the movies” and B= “buying popcorn”. Mira’s statement can be written as the implication  $A \Rightarrow B$ .

Was A true in Mira’s case?

Was B true in Mira’s case?

Does what happened disprove Mira’s statement?

2. Kira said:

“If I go to the park, then I will ride my bicycle.”

She went to the park, but forgot her bicycle at home. Was she truthful in her earlier statement? Why or why not?

What is A? What is B? Write down the implication.

Was A true in Kira’s case?

Was B true in Kira’s case?

Does what happened disprove Kira’s statement?

3. Vera said:

“If I brush my teeth, then I will not get a cavity.”

She did not brush her teeth and she got a cavity. Does her experience mean that brushing teeth does not prevent cavities? Why or why not?

What is A? What is B? Write down the implication.

Was A true in Vera's case?

Was B true in Vera's case?

Does what happened disprove Vera's statement?

4. Using your answers for problems 1-3, fill in the table below for the value (true or false) of  $A \Rightarrow B$ .

A	B	$A \Rightarrow B$
True	True	True
False	True	_____
True	False	_____
False	False	_____

5. Sarah wants to prove that any whole number times the number after it is even. Write down her statement as an "If..., then..." statement.

Dinah tells her: but  $3 \cdot 5$  is odd. Is this a good argument? Why or why not?

What is A and what is B in this problem? Write down the implication.

The *converse* of an implication  $A \Rightarrow B$  is the statement in opposite order,  $B \Rightarrow A$ .

For example, “If I learned something new, then I went to Math Circle today”, is the converse of the implication above.

The converse can either be true or false. Let’s examine this now:

6. For each implication, state whether it is true or false. Then write the converse and decide if it is true or false:

“If an animal is a cow, then it has four legs”

“If today is Sunday, then tomorrow is Monday”

“If a ripe berry is red, then it is a strawberry”

7. What do you notice about converses? Can the original implication be true and the converse be false? Can it be the other way around?

We now try to “reverse the arrow” in  $A \Rightarrow B$  in such a way that we get an equivalent statement.

8. “If today is Sunday, then tomorrow is Monday”.  
Your mom tells you that tomorrow is not Monday. What is your conclusion?

“If a child is in the 3<sup>rd</sup> grade or order, then he or she has learned the multiplication table.”

Bill didn’t learn the multiplication table yet. What is your conclusion?

“If a polygon has 4 vertices, then it has 4 edges.”

Bobby counted 5 edges in a polygon. What is your conclusion?

“If a child has a sister, then he or she has a brother.”

Sandra does not have a brother. Using the statement, what is your conclusion? How is this example different from the others?

9. Let's say  $A = \text{"today is Sunday"}$  and  $B = \text{"tomorrow is Monday"}$ . We can say "tomorrow is not Monday" is "not B". Then what is "not A"?

How can we relate "not B" to "not A"?

Is it true that "not A"  $\Rightarrow$  "not B"?

10. Let's say  $A = \text{"The animal is a cow"}$  and  $B = \text{"The animal has 4 legs"}$ .

What is "not A"?

What is "not B"?

Is it true that "not B"  $\Rightarrow$  "not A"?

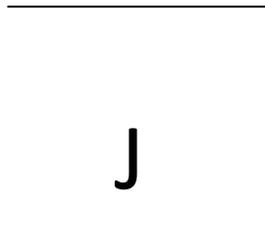
Is it true that "not A"  $\Rightarrow$  "not B"?

## Mirror Problems

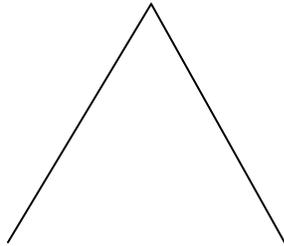
11. Jason has two mirrors that he puts together to form a corner.



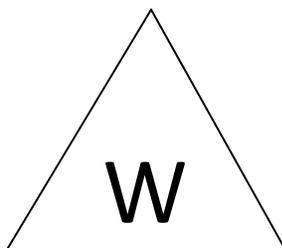
He puts the letter "J" next to the mirror. Draw the reflections that result.



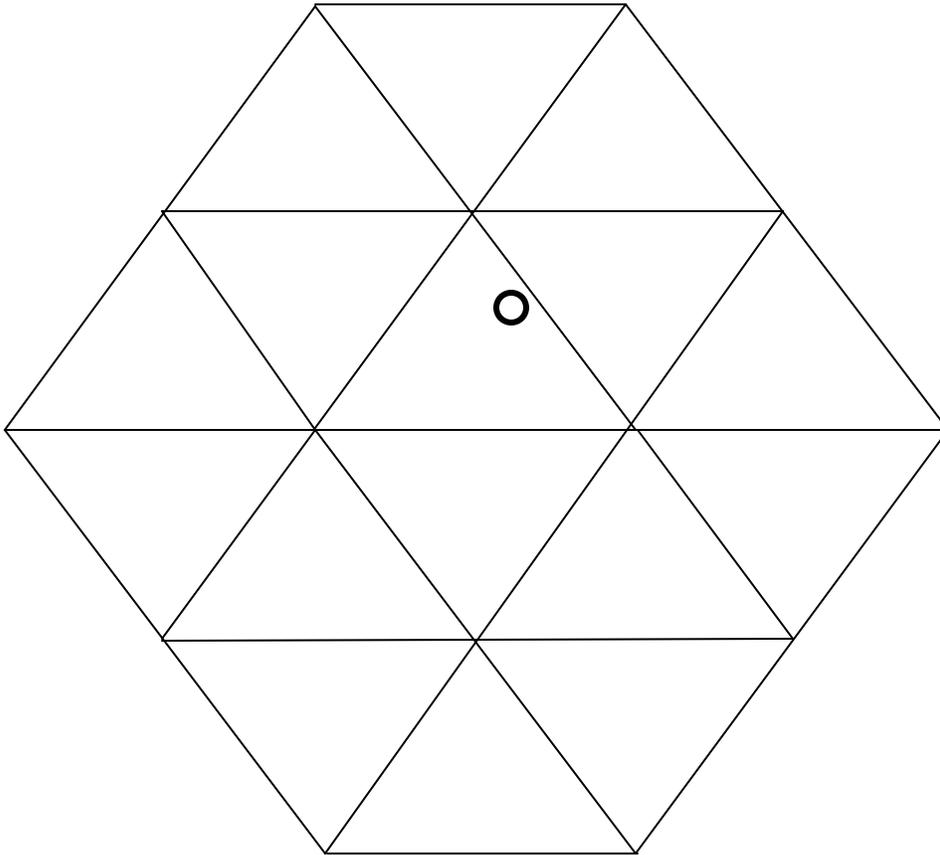
12. Wanda now takes two mirrors and shapes them into a corner:



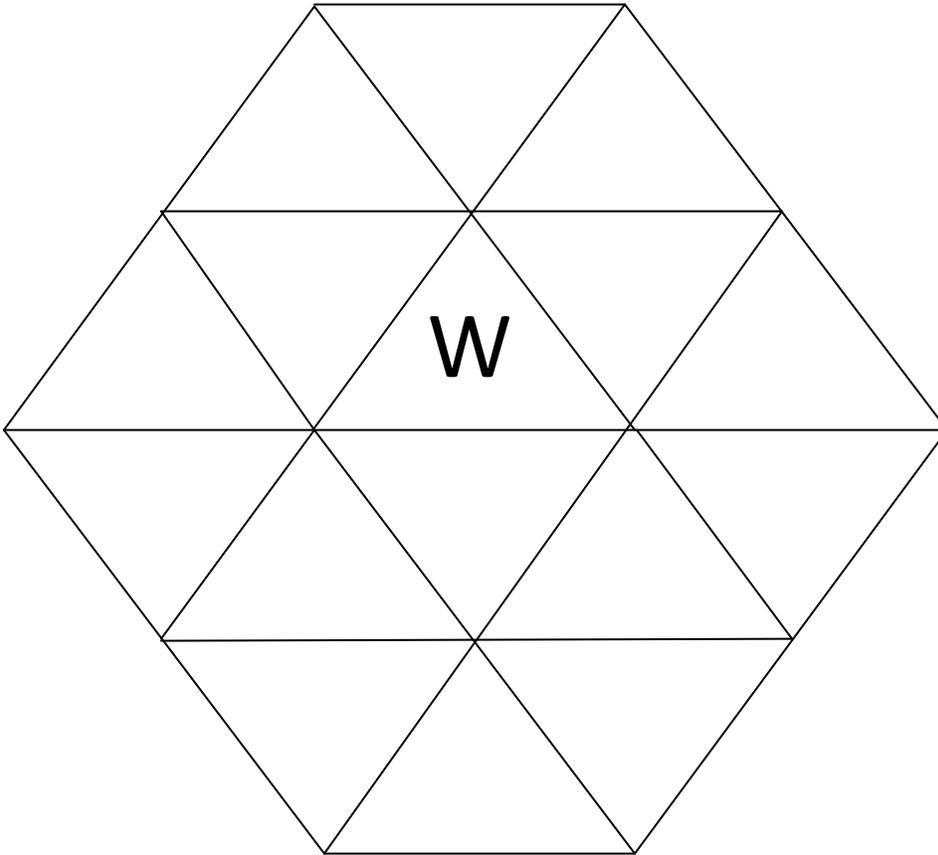
She puts the letter W inside the mirror. Draw the reflections that result.



13. Now we have many mirrors in the same way that Wanda placed them.  
Draw the reflections that result for the image below.



14. Wanda now places a W inside the mirrors. Draw the reflections that result for the image below.



What does this remind you of?