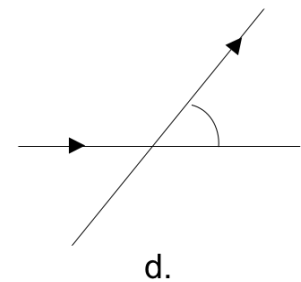
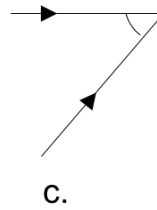
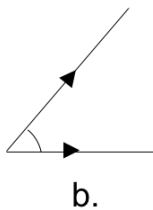
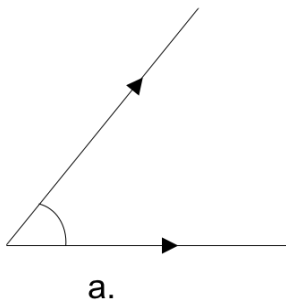


Math Circle
Beginners Group
May 8, 2016
Geometry

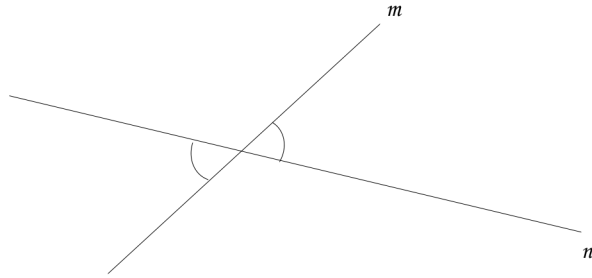
Warm-up Problem: Grandfather and Grandson

1. “In 1932, I was as old as the last two digits of my birth year. When I mentioned this interesting coincidence to my grandfather, he surprised me by saying that the same applied to him also. I thought that was impossible...”
“Of course that’s possible,” a young woman said.
“Believe me, it’s quite possible and grandfather proved it too. How old was each of us in 1932?”

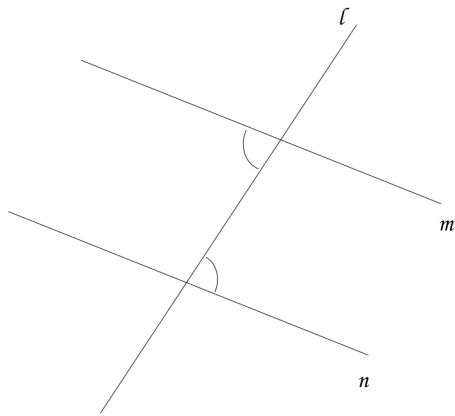
2. Identify which of the following angles are equal, given that all the horizontal lines are parallel to each other and all the slanted lines are parallel to each other.



3. Lines m and n intersect each other. Write a geometric proof in your own words to show that the two angles shown are equal to each other.



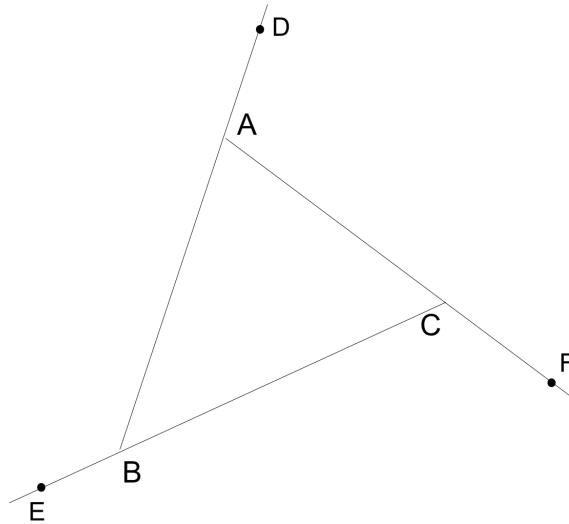
4. Line l intersects two parallel lines m and n . Write a geometric proof in your own words to show that the two angles shown are equal to each other.



Exterior Angles

The sides of a triangle come together at vertices and form **interior angles**. These angles, like the name suggests, lie in the interior of the triangle.

An **exterior angle** is formed outside the triangle by one side and the extension of an adjacent side. Look at the figure below.

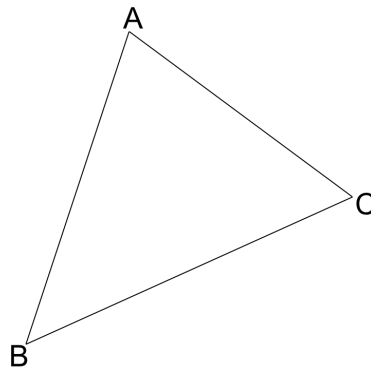


1. In $\triangle ABC$ above, name the interior and exterior angles of the triangle.

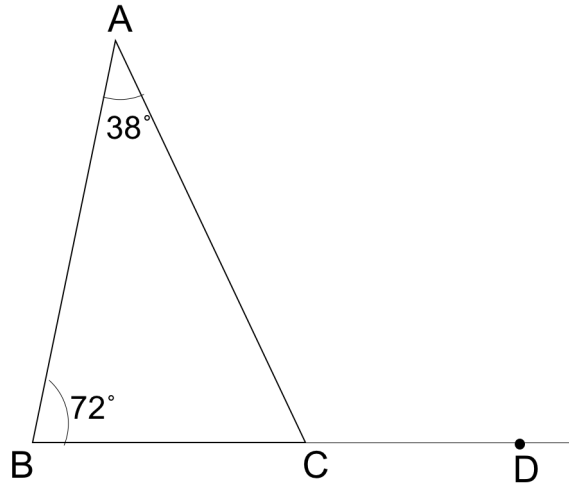
Interior angles:

Exterior angles:

2. Draw exterior angles different from the ones shown above at each of the vertices A , B , and C .

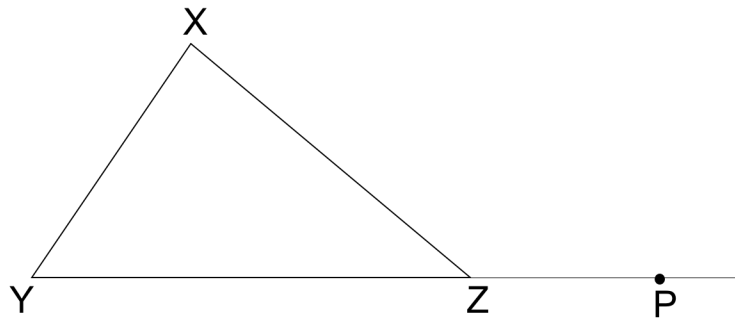


3. Find $\angle ACD$ in the following figure.



This straightforward solution suggests that we can prove something general about exterior angles in triangles.

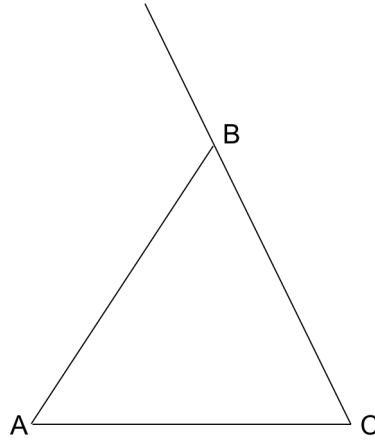
4. Prove that $\angle X + \angle Y = \angle XZP$ in the diagram shown below.



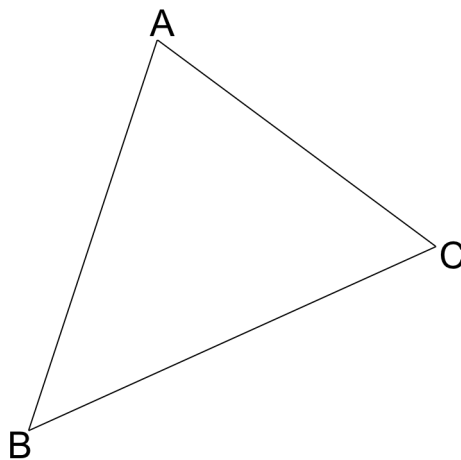
Let us call $\angle X$ and $\angle Y$ the remote interior angles of exterior angle $\angle XZP$ in $\triangle XYZ$ in order to distinguish them from $\angle XZY$. So, we can write down the following property:

Any exterior angle of a triangle is equal to the sum of its remote interior angles.

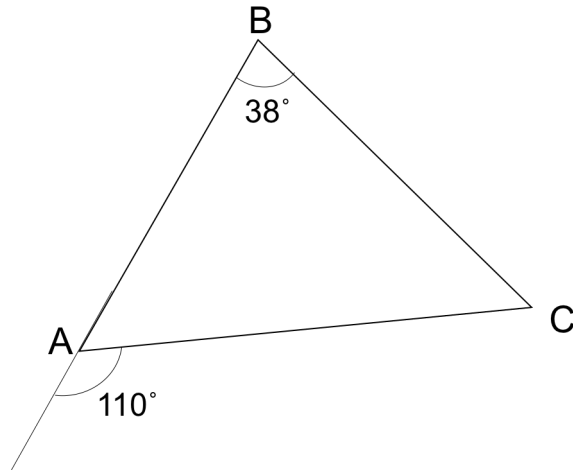
5. Divide the exterior angle at B shown in the picture into two angles equal to the interior angles at A and C .



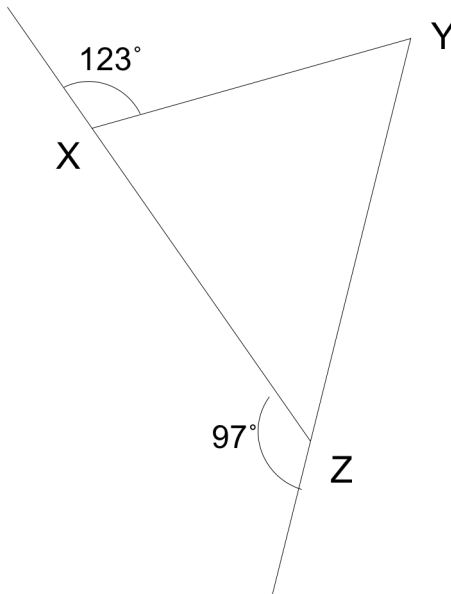
6. Prove that the sum of angles in a triangle is equal to 180° . (Hint: Draw a line parallel to BC passing through vertex A .)



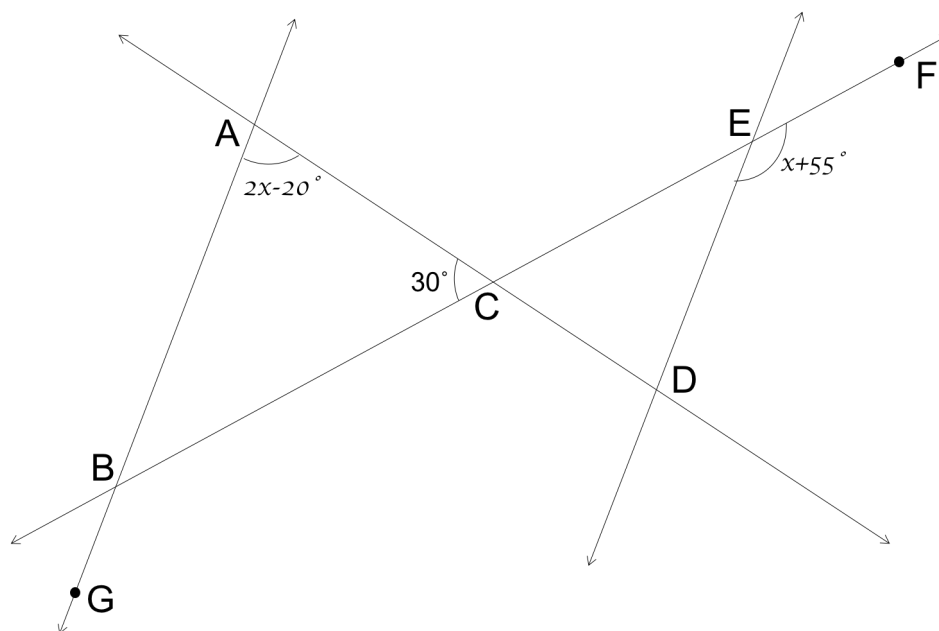
7. Find $\angle ACB$ in the figure below using the exterior angle property.



8. Find $\angle XYZ$ in the figure below using the exterior angle property.

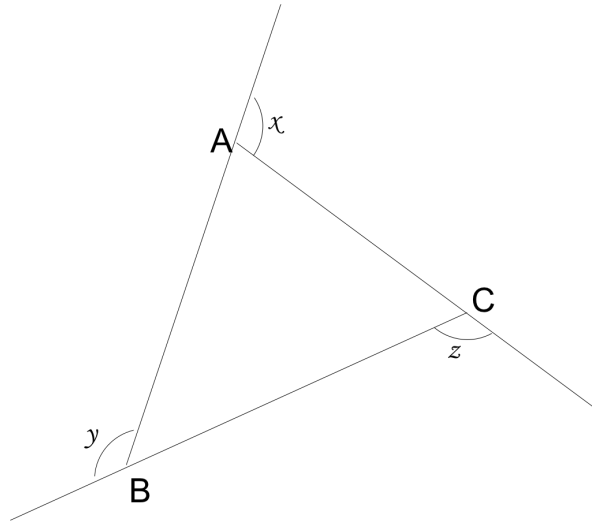


9. In the diagram, $AB \parallel DE$, $\angle BAC = 2x - 20^\circ$, $\angle ACB = 30^\circ$, and $\angle DEF = x + 55^\circ$. Find $\angle CED$.¹



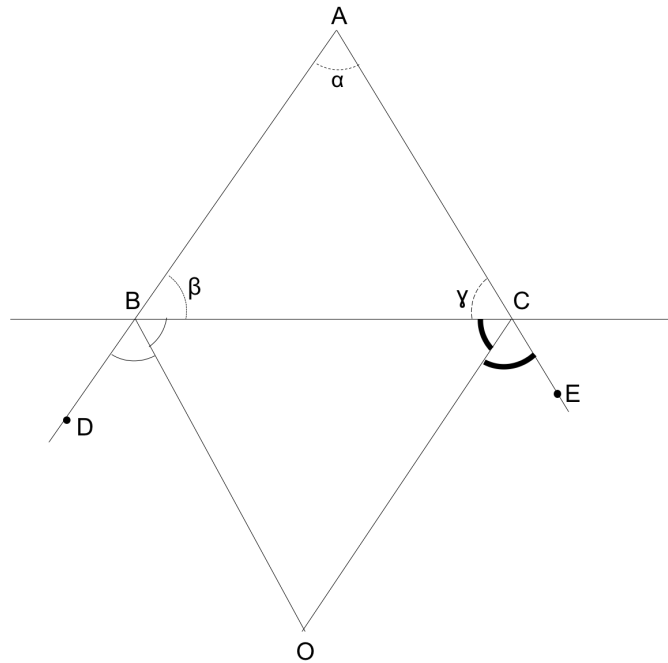
¹The problem is taken from The Art of Problem Solving Introduction to Geometry by Richard Rusczyk.

10. Find the sum of the angles: $x + y + z$.



11. Must the exterior angle of a triangle be always greater than 90° ? Why or why not? Give examples.

12. In the figure, segment BO bisects exterior angle $\angle DBC$. Segment CO bisects exterior angle $\angle ECB$. $\angle A = \alpha$, $\angle ABC = \beta$, and $\angle ACB = \gamma$. Show that $\angle BOC = 90^\circ - \frac{1}{2}\angle A$.



- (a) Compute $\angle DBC$ using the external angle property. What is $\angle OBC$?
- (b) Compute $\angle ECB$ using the external angle property. What is $\angle OCB$?
- (c) Compute $\angle OBC + \angle OCB$ and express it in terms of α .
- (d) Compute $\angle BOC$.

- (b) What is the sum of the measures of all the interior angles in an n -gon?
- (c) A polygon in which all sides are equal is called a **regular polygon**. What is the measure of each angle in a regular n -gon?
- (d) What is sum of interior angles in a 12-sided polygon?
4. What is the sum of the exterior angles in a pentagon? Draw a figure showing all the exterior angles.

(a) In any pentagon, would all exterior angles measure the same?

(b) What is the measure of each exterior angle in regular pentagon?

5. What is the sum of exterior angles in any polygon? Give a geometric proof to support your answer.
6. Find out the measure of each interior and exterior angle in the following regular polygons.
- (a) a heptagon (7-sided polygon)

 - (b) a nonagon (9-sided polygon)
7. Find the number of sides in a regular polygon in which each interior angle measures 144° . What is this polygon called?