

The Island of Knights and Liars

Warm-up problems

1. The age of Peter's great grandfather is the smallest three digit number written with three *different* digits. How old is Peter's great grandfather?

102

2. Tara was prescribed a medication that she has to take every 30 minutes. She took the first pill at 1pm and has to take a total of 6 pills. When will she take the last one?

3:30 pm

3. It is raining at midnight on Tuesday. Do you think we can we expect sunny weather in 72 hours?

no, we will not expect sunny weather, because 72 hours later will be at midnight

The Island of Knights and Liars

Island of Knights and Liars

There are two types of inhabitants on the *Island of Knights and Liars*:

- *Knights* always tell the truth;
- *Liars* always lie;

Occasionally, *tourists* also visit the island. Tourists sometimes lie and sometimes tell the truth.

1. You ask two people you meet on the Island, Tom and Tim, if they are Knights.
 - Tom says he is a Knight. Can you figure out who Tom is?

no — Knights, Liars, and Tourists can all say that they are a Knight

- Tim says he is a Liar. Can you figure out who Tim is?

Cannot be a Knight — Knights cannot lie
Cannot be a Liar — Liars would not tell the truth
Must be a Tourist

2. You encounter three Islanders, Mathew, Jack, and Kate. You ask Mathew if he is a Knight.
 - Mathew answers the question, but he mumbles and you can't hear his answer.
 - You ask Jack what Mathew said. Jack says that Mathew said he was a Liar.
 - Kate says, "Don't believe Jack, he is a Liar!"

Who are these Islanders? (Note: they are not tourists.)

Jack claims that Mathew said he was a Liar, but that is impossible (Knights won't say they are a Liar, Liars cannot tell the truth, and there are no tourists). We thus know that Jack is lying, so Jack is a Liar. Kate warns us that Jack is a Liar, so Kate is telling the truth and is thus a Knight.

We know from Jack's lie that Mathew said "I am a Knight." From this statement, we can't tell if Mathew is a Knight or a Liar, as Knights will say they are Knights and Liars will say they are Knights.

Mathew: either a Knight or a Liar

Jack: Liar

Kate: Knight

3. While visiting the Knights and Liars Island, I had a conversation with a local Knight. I asked him the same question twice, and he gave me two different answers. What could be my question?

Possible answers:

“What time is it?”

“What are you thinking about right now?”

4. Andy and Ben live on the *Island of Knights and Liars*. Andy says: “We are both liars”. Who is Andy and who is Ben? (Note: since they live on the island, they are not tourists.)

They cannot both be liars — Andy is lying.

Andy: Liar

Ben: Knight

5. Alice and Bob live on the *Island of Knights and Liars*.

- Alice says: “We are the same kind”;
- Bob says: “We are different kinds”.

Who is Alice and who is Bob?

Alice: Liar

Bob: Knight

Cryptarithms are mathematical puzzles in which the digits are replaced by letters of the alphabet. Remember that there are 10 digits: 0, 1, ..., 8, 9. Moreover, each letter represents the same digit throughout the problem.

To solve a cryptarithm means to find what digits correspond to what letters so that you get a valid mathematical equality.

Solve the following cryptarithms. (Note that sometimes several solutions are possible.)

$$1. \begin{array}{r} E \ G \ G \\ + \ E \ G \ G \\ \hline P \ A \ G \ E \end{array}$$

$$E = \quad G = \quad A =$$

Note that there can be more than one solution. This is one possible solution.

$$E=8, G=9, A=7, P=1$$

$$2. \begin{array}{r} S \ H \ E \\ + \ E \ E \ L \\ \hline E \ L \ S \ E \end{array}$$

$$S = \quad H = \quad E = \quad L =$$

Note that there can be more than one solution.
This is one possible solution.

$$S=9, H=8, E=1, L=0$$

$$3. \begin{array}{r} M \ O \ M \\ + \ M \ O \ P \\ \hline Y \ O \ Y \ O \end{array}$$

$$M = \quad O = \quad Y = \quad P =$$

Note that there can be more than one solution.
This is just one possible solution.

$$M=7, O=5, Y=1, P=8$$

Fun problems

1. Kara noticed that the sum of her age, her mom's age and her dad's age equals to 70. She is wondering when the sum of their ages will be equal to 100. Can you help her?

10 years later

2. Andy took a number, added 1 to it, then subtracted 2, then multiplied by 3, and then divided by 4. The result was 6. What number did Andy start with?

The number 9

3. Five apples cost more than six bananas. What is more expensive: six apples or seven bananas?

6 apples

4. Sarah multiplied all the numbers starting with 3 and ending with 23. What is the last digit of the result? (*Hint:* Please do not multiply all these numbers!)

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