

# The Island of Knights and Liars

## Warm-up problems

1. The age of Peter's great grandfather is the smallest three digit number written with three *different* digits. How old is Peter's great grandfather?
2. Tara was prescribed a medication that she has to take every 30 minutes. She took the first pill at 1pm and has to take a total of 6 pills. When will she take the last one?
3. It is raining at midnight on Tuesday. Do you think we can we expect sunny weather in 72 hours?

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## Island of Knights and Liars

There are two types of inhabitants on the *Island of Knights and Liars*:

- *Knights* always tell the truth;
- *Liars* always lie;

Occasionally, *tourists* also visit the island. Tourists sometimes lie and sometimes tell the truth.

1. You ask two people you meet on the Island, Tom and Tim, if they are Knights.
  - Tom says he is a Knight. Can you figure out who Tom is?
  
  - Tim says he is a Liar. Can you figure out who Tim is?
  
2. You encounter three Islanders, Mathew, Jack, and Kate. You ask Mathew if he is a Knight.
  - Mathew answers the question, but he mumbles and you can't hear his answer.
  - You ask Jack what Mathew said. Jack says that Mathew said he was a Liar.
  - Kate says, "Don't believe Jack, he is a Liar!"

Who are these Islanders? (Note: they are not tourists.)

3. While visiting the Knights and Liars Island, I had a conversation with a local Knight. I asked him the same question twice, and he gave me two different answers. What could be my question?
  
4. Andy and Ben live on the *Island of Knights and Liars*. Andy says: “We are both liars”. Who is Andy and who is Ben? (Note: since they live on the island, they are not tourists.)
  
  
  
  
  
  
  
  
  
  
5. Alice and Bob live on the *Island of Knights and Liars*.
  - Alice says: “We are the same kind”;
  - Bob says: “We are different kinds”.

Who is Alice and who is Bob?

*Cryptarithms* are mathematical puzzles in which the digits are replaced by letters of the alphabet. Remember that there are 10 digits: 0, 1, ..., 8, 9. Moreover, each letter represents the same digit throughout the problem.

To solve a cryptarithm means to find what digits correspond to what letters so that you get a valid mathematical equality.

Solve the following cryptarithms. (Note that sometimes several solutions are possible.)

$$1. \begin{array}{r} E \ G \ G \\ + \ E \ G \ G \\ \hline P \ A \ G \ E \end{array}$$

$$E = \quad G = \quad A =$$

$$2. \begin{array}{r} S \ H \ E \\ + \ E \ E \ L \\ \hline E \ L \ S \ E \end{array}$$

$$S = \quad H = \quad E = \quad L =$$

$$3. \begin{array}{r} M \ O \ M \\ + \ M \ O \ P \\ \hline Y \ O \ Y \ O \end{array}$$

$$M = \quad O = \quad Y = \quad P =$$

## Fun problems

1. Kara noticed that the sum of her age, her mom's age and her dad's age equals to 70. She is wondering when the sum of their ages will be equal to 100. Can you help her?
2. Andy took a number, added 1 to it, then subtracted 2, then multiplied by 3, and then divided by 4. The result was 6. What number did Andy start with?
3. Five apples cost more than six bananas. What is more expensive: six apples or seven bananas?
4. Sarah multiplied all the numbers starting with 3 and ending with 23. What is the last digit of the result? (*Hint:* Please do not multiply all these numbers!)