

## Take Away Games

April 17, 2016

### Warmup

1. Write the name of a two-player game.
2. Describe the strategy you would use to win the game against your opponent.
3. Is there more than one strategy that can be used to win? Justify your answer.

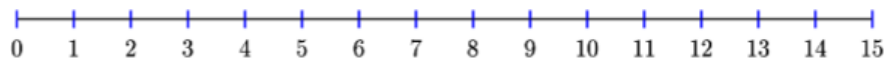
## Last Ball Losses

- Two people are playing a game where there is a pile of 15 balls. Players 1 and 2 take turns taking a ball (or balls) from the pile, with Player 1 always going first. If you end up taking the last ball, you **lose**.

(a) For part (a), you can only take one ball at a time.

i. In order to win, should you be Player 1 or Player 2?

- ii. On the number line below, write either a W or an L on top of each number to show whether it is a winning position or a losing position if it is the opponent's turn.

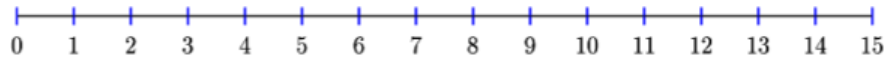


iii. What is the pattern?

(b) For part (b), you can only take two balls at a time.

i. In order to win, should you be Player 1 and Player 2?

- ii. On the number line below, write either a W or an L on top of each number to show whether it is a winning position or a losing position if it is the opponent's turn.

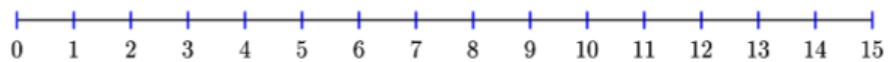


- iii. What is the pattern?

(c) For part (c), you can only take three balls at a time.

- i. In order to win, should you be Player 1 and Player 2?

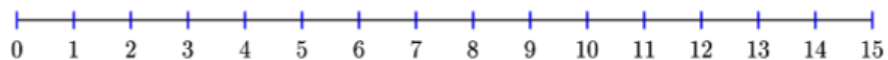
- ii. On the number line below, write either a W or an L on top of each number to show whether it is a winning position or a losing position if it is the opponent's turn.



- iii. What is the pattern?

2. Now you can take 1 or 2 ball(s) at a time. All other rules remain the same.

- (a) On the number line below, write either a W or an L on top of each number to show whether it is a winning position or a losing position if it is the opponent's turn.



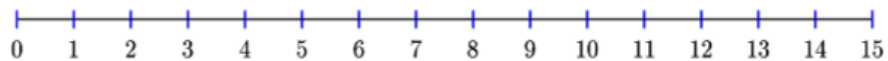
- (b) What is the pattern?

- (c) In order to win, should you be Player 1 or Player 2?

- (d) How many balls should you take your first turn?

3. Now you can take 1 or 3 ball(s) at a time per turn. All other rules remain the same.

- (a) On the number line below, write either a W or an L on top of each number to show whether it is a winning position or a losing position if it is the opponent's turn.



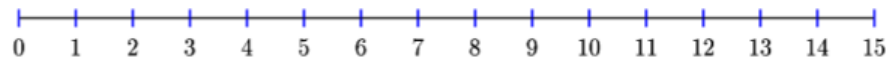
(b) What is the pattern?

(c) In order to win, should you be Player 1 or Player 2?

(d) How many balls should you take your first turn?

4. Now you can take 2 or 3 balls at a time per turn. All other rules remain the same.

(a) On the number line below, write either a W or an L on top of each number to show whether it is a winning position or a losing position if it is the opponent's turn.



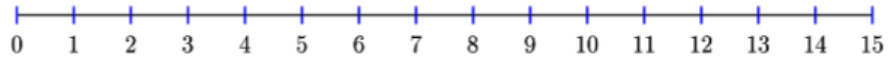
(b) What is the pattern?

(c) In order to win, should you be Player 1 or Player 2?

(d) How many balls should you take your first turn?

5. Now you can take 1, 2 or 3 ball(s) at a time per turn. All other rules remain the same.

(a) On the number line below, write either a W or an L on top of each number to show whether it is a winning position or a losing position if it is the opponent's turn.



(b) What is the pattern?

(c) In order to win, should you be Player 1 or Player 2?

(d) How many balls should you take your first turn?

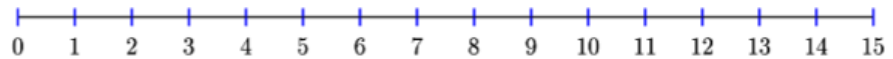
(e) If there are now 16 balls instead of 15, do you still go first and why?

## Last Ball Wins

1. The rules have now changed! Two people are playing a game where there is a pile of 15 balls. Players 1 and 2 take turns taking a ball (or balls) from the pile, with Player 1 always going first. If you end up taking the last ball, you **win**.

- (a) For part (a), you can only take one ball at a time.
  - i. In order to win, should you be Player 1 or Player 2?

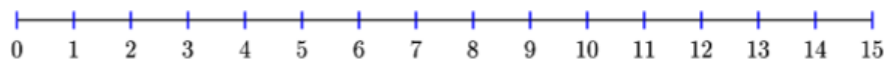
- ii. On the number line below, write either a W or an L on top of each number to show whether it is a winning position or a losing position if it is the opponent's turn.



- iii. What is the pattern?

- (b) For part (b), you can only take two balls at a time.
  - i. In order to win, should you be Player 1 or Player 2?

- ii. On the number line below, write either a W or an L on top of each number to show whether it is a winning position or a losing position if it is the opponent's turn.

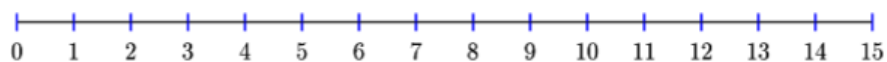


- iii. What is the pattern?

(c) For part (c), you can only take three balls at a time.

- i. In order to win, should you be Player 1 or Player 2?

- ii. On the number line below, write either a W or an L on top of each number to show whether it is a winning position or a losing position if it is the opponent's turn.

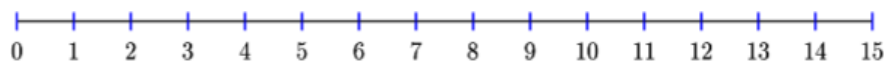


- iii. What is the pattern?



2. You can now take one or two balls per turn.

- (a) On the number line below, write either a W or an L on top of each number to show whether it is a winning position or a losing position if it is the opponent's turn.

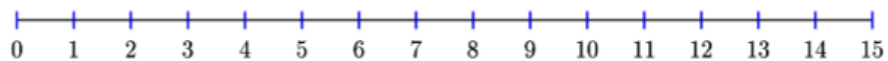


- (b) Should you be Player 1 or Player 2 in order to win?

- (c) How many balls should you take your first turn?

3. You can now take one or three balls per turn.

- (a) On the number line below, write either a W or an L on top of each number to show whether it is a winning position or a losing position if it is the opponent's turn.

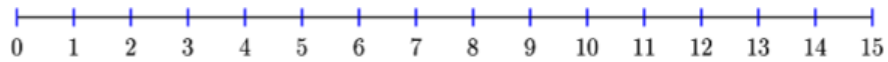


- (b) Should you be Player 1 or Player 2 in order to win?

- (c) How many balls should you take your first turn?

4. You can now take two or three balls per turn.

- (a) On the number line below, write either a W or an L on top of each number to show whether it is a winning position or a losing position if it is the opponent's turn.

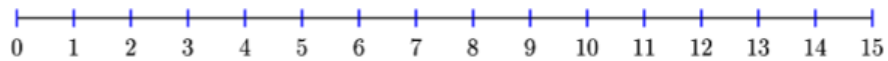


- (b) Should you be Player 1 or Player 2 in order to win?

- (c) How many balls should you take your first turn?

5. You can now take one, two, or three balls per turn.

- (a) On the number line below, write either a W or an L on top of each number to show whether it is a winning position or a losing position if it is the opponent's turn.



- (b) Should you be Player 1 or Player 2 in order to win?

- (c) How many balls should you take your first turn?

- (d) If there are now 16 balls instead of 15, do you still go first and why?

**Activity Time!**

1. Time for some fun! Come up with your own take away game! Choose two numbers that you can take away (i.e. 1 or 2). Write down the rules of your game. Then play against 3 of your classmates who have already completed their worksheets.