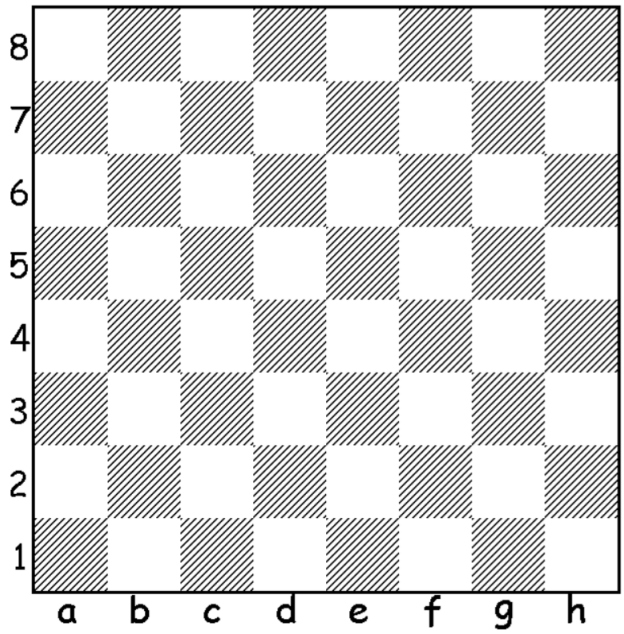
Fun on Chessboard II

Warm Up

Shade in the following squares on the chessboard. What letter does it make?

b2, c2, e6, b7, c5, f5, e5, d4, f7, f4, b5, c4, d7, b3, e7, f6, c7, c3, b4, e4, c6, b6



The letter P

II *Rook Race* Game

Two players are playing the following game:

- **Initial position:** Two rooks are placed on two squares of a chessboard.
- **Move:** Move *any* of the rooks to the right by any number of squares.
- **Goal:** To be the *last* person to reach the rightmost square.

1. Play this game with your partner several times. Try to come up with a winning strategy. That is, come up with a method that allows you to win no matter what your opponent does. Only one of the players (first or second) has a winning strategy. You need to find it. Here are the initial positions:

   (a) Rook I on f3, Rook II on f6
   Which player can win? (**Player II**)

   (b) Rook I on d3, Rook II on d6
   Which player can win? (**Player II**)

   (c) In general, if both Rooks are the same number of squares away from the right edge, which player can always win? How
   Solution: The second player will win. On his first move, player I is forced to move one of the rooks ahead of the other one. As long as Player II makes sure to line the

two rooks up again, this strategy will insure that Player I will reach the right edge first.

2. Now use the following initial positions:

   (a) Rook I on f3, Rook II on d6
   Which player can win? (**Player I**)

   (b) Rook I on e3, Rook II on a6
   Which player can win? (**Player I**)

   (c) Rook I on a3, Rook II on b6
   Which player can win? (**Player I**)

   (d) Rook I on c3, Rook II on g6
   Which player can win? (**Player I**)

   (e) In general, if the rooks are a different number of squares away from the right edge, which player can always win? How?
   Player I can always win, by lining up the two rooks. Once they are lined up, no matter what move Player II makes, Player I can line the rooks back up, thus gaurantee-ing that he will reach the edge of the board second.

II *Put Rook Into the Corner* Game

Two players are playing the following game:

- **Initial position:** One Rook is placed somewhere on the chessboard.
- **Move:** Move the Rook down or left by any number of squares.
- **Goal:** To put the Rook into the left lower corner.

3. Play this game with your partner several times. Try to come up with a winning strategy. That is, come up with a method that allows you to win no matter what your opponent does. In every position, only one of the players (first or second) has a winning strategy. You need to find it. Here are the initial positions:

   (a) Rook on c3. Which player can win?
   (**Player II**)

   (b) Rook on d4. Which player can win?
   (**Player II**)

   (c) Rook on f7. Which player can win?
   (**Player I**)

   (d) Rook on g8. Which player can win?
   (**Player I**)

(e) In general, if the Rook is on the diagonal connecting squares a1 and h8, which player can win? How?

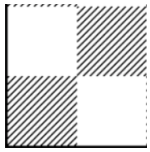
Player II will win in this case. Player I is forced to move the rook away from the diagonal, thus allowing Player II to always bring the rook back to the diagonal. Since the corner piece is on the diagonal, this ensures that Player II will win.

(f) How does the game change if the Rook is placed away from the diagonal? Which player can win now?

Player I can now always win, by using his first move to take the rook to the diagonal. Once it is on the diagonal, the second player must move it away from the diagonal, ensuring that Player I can always bring the rook back to the diagonal, and ultimatley win.

Chessboard Math!

1. Look at the 2x2 square.



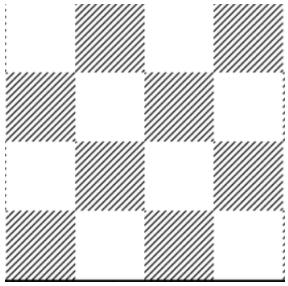
   (a) How many 1x1 squares are white?

   2

   (b) How many 1x1 squares are black?

   2

2. Look at this 4x4 square.



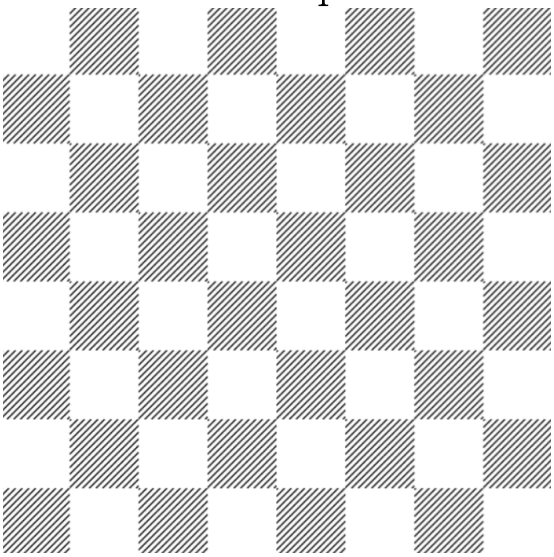
   (a) How many 1x1 squares are white?

   8

   (b) How many 1x1 squares are black?

   8

So far, it is easy to just count the number of squares. But what if we have a very large square like a 6x6 square? It takes a lot more time to count all of white squares, and all of the black squares.



Typo: this should be a 6x6!

Let's find an easier way to do this. Idea: instead of counting the number of black squares, we can divide the large 6x6 square into 2x2 squares.

1. Can you use the same method as last week, by counting the bottom left hand corner square? Why or why not?

   We cannot use the same strategy, because we cannot have overlaps. Overlaps would count the same squares multiple times, and we would effectively over-count the number of black squares.

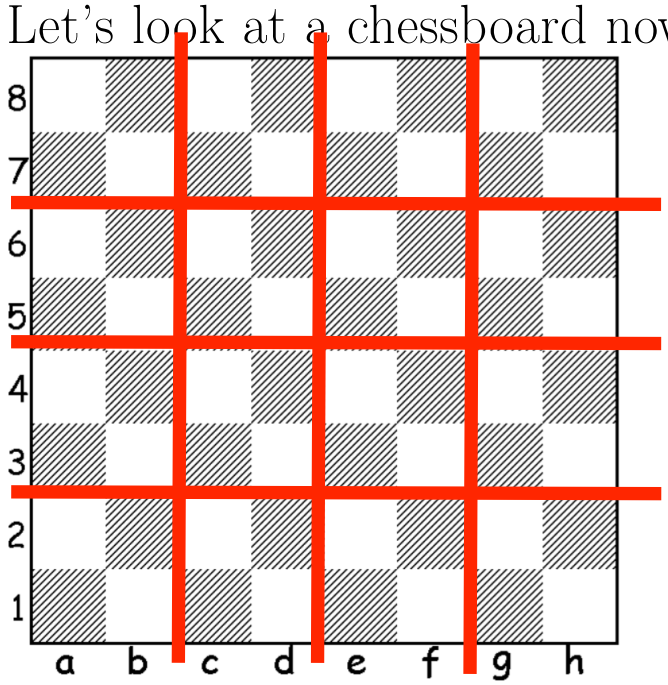
2. How many non-overlapping 2x2 squares are there?

9

(a) We know from before that there are 2 black squares in every 2x2 square. Use this information and your answer above to find the number of black squares on the 6x6 square.

9 x 2 = 18

3. Let's look at a chessboard now.



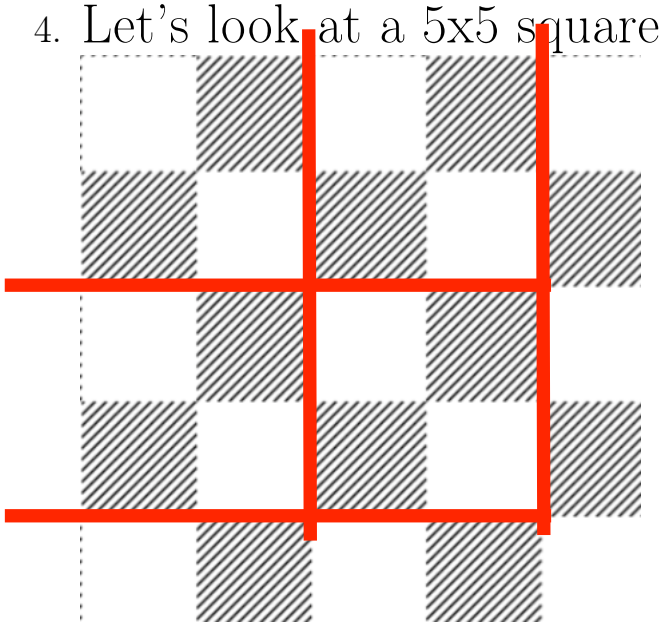
(a) How many total non-overlapping 2x2 squares are there on a chessboard? Draw lines on the chessboard.

16

(b) Use your answer from part (a) and the fact that there are 2 black squares in each 2x2 square to find the total number of black squares.

16x2=32

4. Let's look at a 5x5 square.



(a) Can you divide the 5x5 square completely into 2x2 squares? What happens?

No, you cannot. There will be some leftover squares.

(b) Draw lines to divide the 5x5 square into as many non-overlapping 2x2 squares as you can. How many black squares are there in total in the non-overlapping 2x2 squares?

4 x 2 = 8

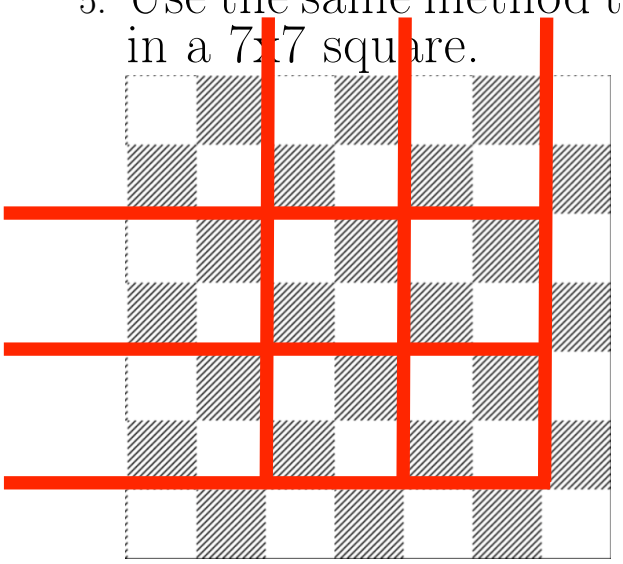
(c) Count the number of black squares that are remaining after all the divisions are made.

4

(d) How many black squares are there in total in a 5x5 square?

8 + 4 =12

5. Use the same method to find the number of black squares in a 7x7 square.



(a) How many non-overlapping 2x2 squares can you draw? How many black squares are in these 2x2 squares?

9 x 2 = 18

(b) Count the number of black squares that are remaining after all the divisions are made.

6

(c) How many black squares are there in total on the 7x7 square?

18 + 6 =24

6. Can you describe in words how to find the number of black squares in a large square? (Hint: Write a separate rule for squares that have an even side length (i.e. 4x4, 6x6, 8x8) and a separate rule for squares that have an odd side length (i.e. 5x5, 7x7, 9x9).

For even side lengths: Divide the board evenly into 2x2 squares, and count the number of 2x2 squares that are available. Then, since each 2x2 square has 2 black squares, multiply the number by 2.

For odd side lengths: Divide the board into as many 2x2 squares as possible. Multiply this number by 2 to get the number of black squares in all of the 2x2 squares. Then, count the remaining squares and add them to the last number to get the total.