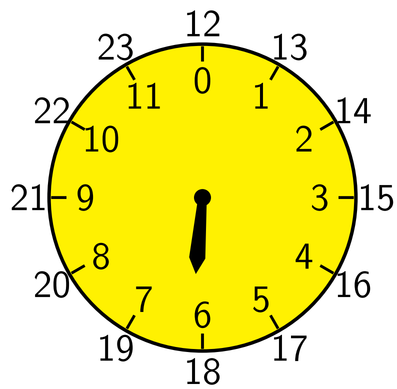
LAMC Junior Circle April 3, 2016

Problem 1. It takes a grandfather's clock 30 seconds to chime 6 o'clock. Assuming that the time of each chime is negligible compared to the time intervals between the chimes, how much time would it take the clock to chime 12?

Clock Arithmetic or a Circle as a Number Line

One way to turn a circle into a number line is to divide it into twelve equal parts. In this case, one step is usually called one hour.



0 coincides with 12. The hour hand moves from 0 to 1, from 1 to 2, ... from 11 to 12 just as it would have on the straight number line. However, 12 equals 0 on this circle, so there it goes again, from 1 to 2, and so on. We write down the fact that 12 equals 0 as

$$12 \equiv 0 \ (mod\ 12) \tag{0.1}$$

and read it as *12 is congruent to 0 modulo 12.* The usual "=" sign is reserved for the straight number line; we use "≡" on the circle instead. The *mod* 12 symbol tells us that the circle is divided into 12 equal parts, so 12 coinsides with 0, 13 – with 1, 14 – with 2, and so on. Or in the new notations,

$$13 \equiv 1 \ (mod\ 12), \ \ 14 \equiv 2 \ (mod\ 12), \ldots, 23 \equiv 11 \ (mod\ 12),$$

$$24 \equiv 12 \equiv 0 \ (mod\ 12).$$

Problem 2. Divide the following numbers by 12, and write the remainders of the divisions.

$$15 \div 12 = \_\_R\_\_$$

$$21 \div 12 = \_\_R\_\_$$

$$37 \div 12 = \_\_R\_\_$$

$$46 \div 12 = \_\_R\_\_$$

$$80 \div 12 = \_\_R\_\_$$

Now, write down the modular congruencies for the following numbers.

$$15\ (mod\ 12) \equiv$$

$$21\ (mod\ 12) \equiv$$

$$37\ (mod\ 12) \equiv$$

$$46\ (mod\ 12) \equiv$$

$$80\ (mod\ 12) \equiv$$

Problem 3. Write down the modular congruencies for the following additions.

$$9 + 4 \equiv \qquad\qquad (mod\ 12)$$

$$18 + 8 \equiv \qquad\qquad (mod\ 12)$$

When subtracting two numbers ‘a‘ and ‘b‘, finding a - b means finding a number ‘c‘ such that c + b = a.
For example, 5 - 3 =2 because when you add 2 to 3, you get 5.

The same occurs with modular arithmetic.

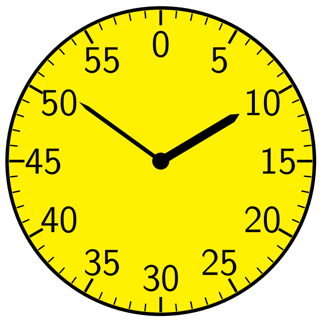
Write down the modular congruencies for the following subtractions.

$$8 - 3 \equiv \qquad\qquad (mod\ 12)$$

$$1 - 11 \equiv \qquad\qquad (mod\ 12)$$

$$4 - 15 \equiv \qquad\qquad (mod\ 12)$$

Another standard way to turn a circle into a number line is to divide it into 60 equal parts. Depending on the situation, the unit step is called either a minute or a second.



All the numbers living on this number line are considered modulo 60. In particular, $60 \equiv 0$ $(mod\ 60)$. There are 60 minutes in an hour.

Problem 4.

$$72 \equiv \qquad\qquad (mod\ 60)$$

$$135 \equiv \qquad\qquad (mod\ 60)$$

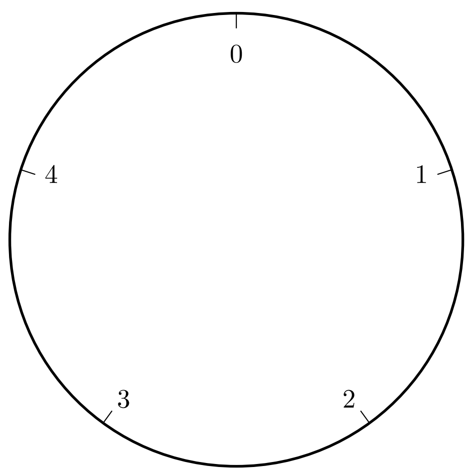
$$55 + 55 \equiv \qquad\qquad (mod\ 60)$$

$$-15 \equiv \qquad\qquad (mod\ 60)$$

$$240 - 59 \equiv \qquad\qquad (mod\ 60)$$

Problem 5. An experiment in a biological lab starts at 7:00 AM and runs for 80 hours. What time will it end?

In the following problems, we will consider the *mod* 5 arithmetic, that of a circle divided into five equal parts.



Problem 6. Write down the modular congruencies for the following multiplation problems.

$$2 \times 3 \equiv \qquad\qquad (mod\ 5)$$

$$4 \times 4 \equiv \qquad\qquad (mod\ 5)$$

$$5 \times 7 \equiv \qquad\qquad (mod\ 5)$$

Similar to how subtraction is related to addition, division is related to multiplcation. Finding $a \div b$ means finding a number 'c' such that
$b \times c = a$.

For example, $6 \div 3 = 2$ because $3 \times 2 = 6$.

Write down the modular congruencies for the following division problems.

$$1 \div 2 \equiv \qquad\qquad (mod\ 5)$$

$$1 \div 3 \equiv \qquad\qquad (mod\ 5)$$

$$1 \div 4 \equiv \qquad\qquad (mod\ 5)$$

Problem 7. Write down the modular congruencies using "mod <u>(*your age*)</u>".

$$11 + 39 \equiv \qquad\qquad (mod\ \_\_)$$

$$22 + 17 \equiv \qquad\qquad (mod\ \_\_)$$

$$33 - 11 \equiv \qquad\qquad (mod\ \_\_)$$

$$2 - 50 \equiv \qquad\qquad (mod\ \_\_)$$

$$1 \times 10 \equiv \qquad\qquad (mod\ \_\_)$$

$$4 \times 4 \equiv \qquad\qquad (mod\ \_\_)$$