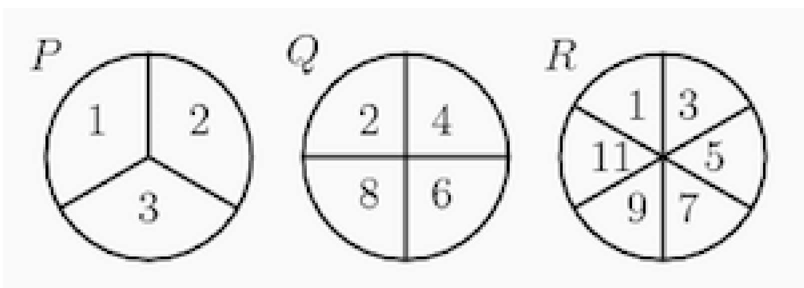
# Winter Quarter Competition

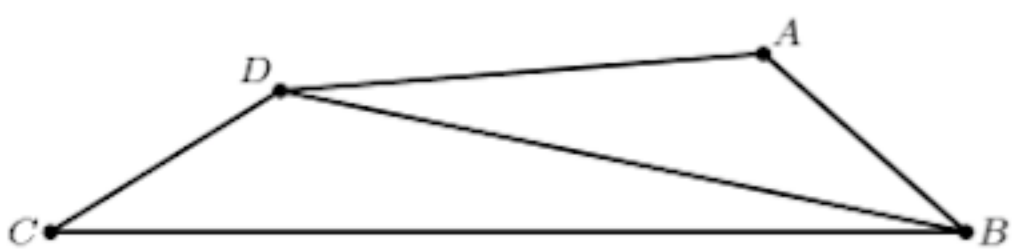
LA Math Circle (Advanced)

March 13, 2016

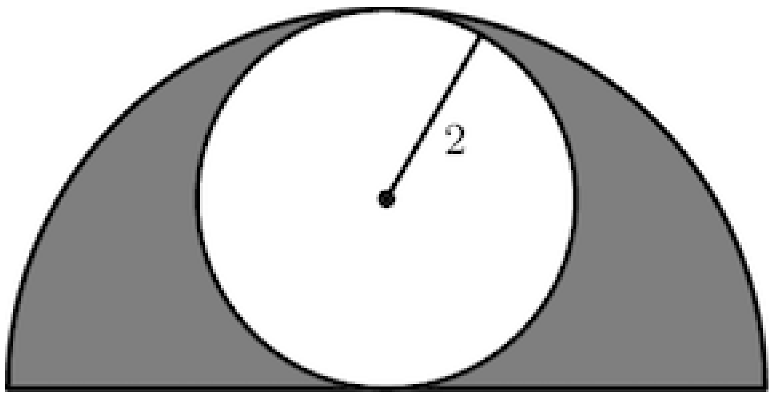
**Problem 1** Jeff rotates spinners $P, Q$, and $R$ and adds the resulting numbers. What is the probability that his sum is an odd number?



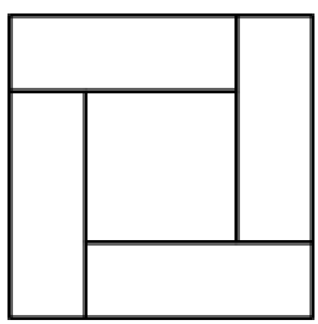

**Problem 2** In quadrilateral $ABCD$, $AB = 5$, $BC = 17$, $CD = 5$, $DA = 9$, and $BD$ is an integer. What is $BD$?



**Problem 3** A circle of radius 2 is inscribed in a semicircle, as shown. The area inside the semicircle but outside the circle is shaded. What fraction of the semicircle's area is shaded?




**Problem 4** Four congruent rectangles are placed as shown. The area of the outer square is 4 times that of the inner square. What is the ratio of the length of the longer side of each rectangle to the length of its shorter side?

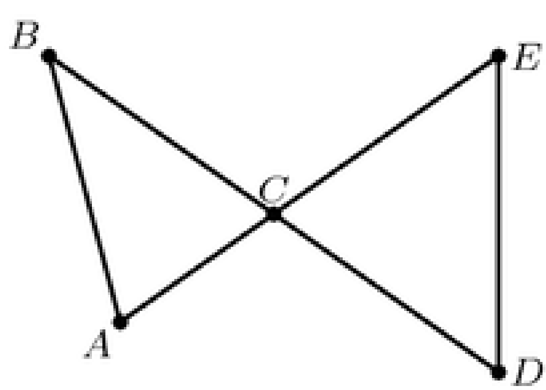


**Problem 5** A flagpole is originally 5 meters tall. A hurricane snaps the

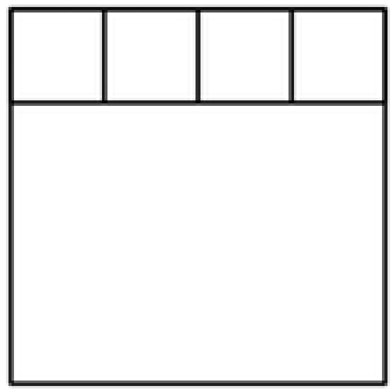
flagpole at a point $x$ meters above the ground so that the upper part, still attached to the stump, touches the ground 1 meter away from the base. What is $x$?

**Problem 6** A rectangular yard contains two flower beds in the shape of congruent isosceles right triangles. The remainder of the yard has a trapezoidal shape, as shown. The parallel sides of the trapezoid have lengths 15 and 25 meters. What fraction of the yard is occupied by the flower beds?

**Problem 7** A Segment $BD$ and $AE$ intersect at $C$ , as shown. $AB = BC = CD = CE$, and $\angle A = \frac{5}{2}\angle B$ . What is the degree measure of $\angle D$?



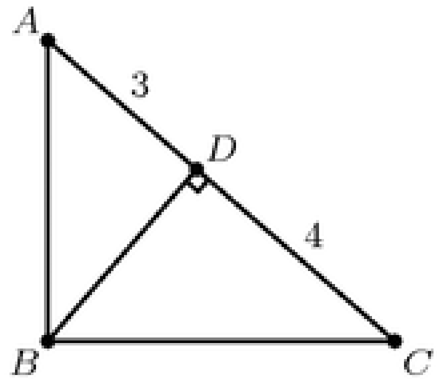
**Problem 8** Four identical squares and one rectangle are placed together to form one large square as shown. The length of the rectangle is how many times as large as its width?



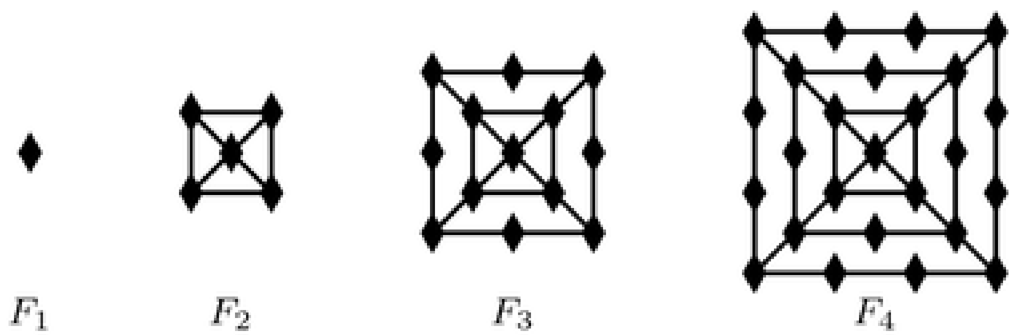
**Problem 9** How many whole numbers are between $\sqrt{8}$ and $\sqrt{80}$?

**Problem 10** Simplify the ratio the $\dfrac{2^{2001} \cdot 3^{2003}}{6^{2002}}$.

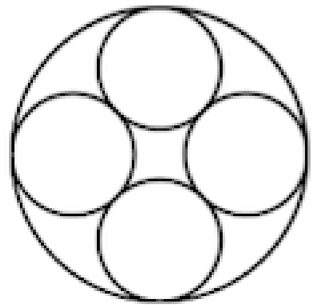
**Problem 11** Triangle $ABC$ has a right angle at $B$. Point $D$ is the foot of the altitude from $B$. $\overline{AD} = 3$ and $\overline{DC} = 4$. What is the area of triangle $ABC$?




**Problem 12** The figures $F_1$, $F_2$, $F_3$, and $F_4$ shown are the first in a sequence of figures. For $n \geq 3$ , $F_n$ is constructed from $F_{n-1}$ by surrounding it with a square and placing one more diamond on each side of the new square than $F_{n-1}$ had on each side of its outside square. For example, figure $F_3$ has 13 diamonds. How many diamonds are there in figure $F_{20}$ ?

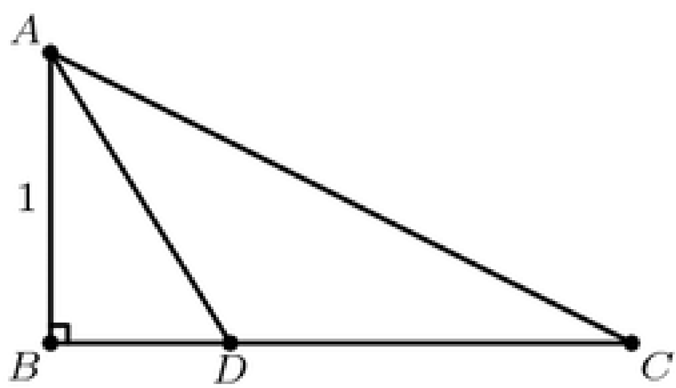



**Problem 13** Many Gothic cathedrals have windows with portions containing a ring of congruent circles that are circumscribed by a larger circle. In the figure shown, the number of smaller circles is four. What is the ratio of the sum of the areas of the four smaller circles to the area of the larger circle?

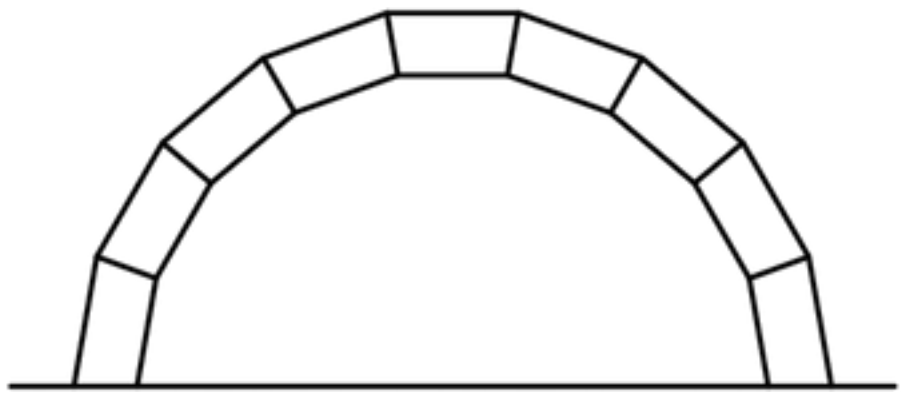


**Problem 14** Points $A$ and $C$ lie on a circle centered at $O$ , each of $\overline{BA}$ and $\overline{BC}$ are tangent to the circle, and $\triangle ABC$ is equilateral. The circle intersects $\overline{BO}$ at $D$ . What is $\frac{BD}{BO}$?

**Problem 15** Triangle $ABC$ has a right angle at $B$, $AB = 1$, and $BC = 2$ . The bisector of $\angle BAC$ meets $\overline{BC}$ at $D$. What is $BD$?



**Problem 16** The keystone arch is an ancient architectural feature. It is composed of congruent isosceles trapezoids fitted together along the non-parallel sides, as shown. The bottom sides of the two end trapezoids are horizontal. In an arch made with 9 trapezoids, let $x$ be the angle measure in degrees of the larger interior angle of the trapezoid. What is $x$?



**Problem 17** The ratio $\dfrac{10^{2000} + 10^{2002}}{10^{2001} + 10^{2001}}$ is closest to which whole number?

**Problem 18** There are 100 players in a single tennis tournament. The tournament is single elimination, meaning that a player who loses a match is eliminated. In the first round, the strongest 28 players are given a bye, and the remaining 72 players are paired off to play. After each round, the remaining players play in the next round. The match continues until only one player remains unbeaten. Find the total number of matches played.

**Problem 19** Jamal wants to save 30 files onto disks, each with 1.44 MB space. 3 of the files take up 0.8 MB, 12 of the files take up 0.7 MB, and the rest take up 0.4 MB. It is not possible to split a file onto 2 different disks. What is the smallest number of disks needed to store all 30 files?

**Problem 20** Coin $A$ is flipped three times and coin $B$ is flipped four times. What is the probability that the number of heads obtained from flipping the two fair coins is the same?

**Problem 21** A $3 \times 3 \times 3$ cube is made of 27 normal dice. Each die's opposite sides sum to 7. What is the smallest possible sum of all of the values visible on the 6 faces of the large cube?

**Problem 22** What is the units digit of $13^{2003}$?

**Problem 23** What is the probability that a randomly drawn positive factor of 60 is less than 7?

**Problem 24** How many positive cubes divide $3! \cdot 5! \cdot 7!$ ?

**Problem 25** A checkerboard of 13 rows and 17 columns has a number written in each square, beginning in the upper left corner, so that the first row is numbered $1, 2, \ldots, 17$, the second row $18, 19, \ldots, 34$, and so on down the board. If the board is renumbered so that the left column, top to bottom, is $1, 2, \ldots, 13$, the second column $14, 15, \ldots, 26$ and so on across the board, some squares have the same numbers in both numbering systems. Find the sum of the numbers in these squares (under either system).

**Problem 26** Three tiles are marked $X$ and two other tiles are marked $O$. The five tiles are randomly arranged in a row. What is the probability that the arrangement reads $XOXOX$?

**Problem 27** Bernardo randomly picks 3 distinct numbers from the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and arranges them in descending order to form a 3-digit number. Silvia randomly picks 3 distinct numbers from the set $\{1, 2, 3, 4, 5, 6, 7, 8\}$ and also arranges them in descending order to form a 3-digit number. What is the probability that Bernardo's number is larger than Silvia's number?

**Problem 28** Let $S$ be the set of the 2005 smallest positive multiples of 4, and let $T$ be the set of the 2005 smallest positive multiples of 6. How many elements are common to $S$ and $T$?

**Problem 29** For how many positive integers $n$ does $1 + 2 + ... + n$ evenly divide from $6n$?

**Problem 30** The number $25^{64} \cdot 64^{25}$ is the square of a positive integer $N$. Find the sum of the digits of $N$ when $N$ is in decimal representation.

**Problem 31** The number obtained from the last two nonzero digits of 90! is equal to $n$. What is $n$?

**Problem 32** Convex quadrilateral $ABCD$ has $AB = 9$ and$CD = 12$. Diagonals $\overline{AC}$ and $\overline{BD}$ intersect at $E$, $AC = 14$, and $\triangle AED$ and $\triangle BEC$ have equal areas. What is $AE$?