Divisibility

A positive whole number is called *prime* if it is only divisible by two numbers—itself and 1. The first few prime numbers are $2, 3, 5, 7, 11, 13, 17, \ldots$.

For some of the problems (such as the ones about the divisibility tests by 3 and by 9) it is convenient to use the following notation. The number written with "digits" $a, b, c, \ldots, z$ is written as $\underline{abc\ldots z}$.

1. Show that the product of three consecutive numbers is always divisible by 6.

2. (a) Ann has a special device which measures angles of exactly 19°. Can she measure an angle of exactly 1°? If so how? If not, why not?

   (b) Ben has a special device which measures angles of exactly 7°. Can he measure an angle of exactly 71°? If so how? If not, why not?

   (c) Carl has a special device which measures angles of exactly 5°. Can he measure an angle of exactly 71°? If so how? If not, why not?

3. Show that a number written with three identical digits is divisible by 37.

4. Show that a number is divisible by 3 if and only if the sum of its digits is divisible by 3. Note that you need to prove two statements:

   (a) If a number is divisible by 3, then the sum of its digits is divisible by 3;

   (b) If the sum of digits of a number is divisible by 3, then the number is divisible by 3.

5. Formulate and prove a test for divisibility by 9 similar to the test for divisibility by 3 in the previous problem.

6. Find all positive whole numbers n so that the fraction $\frac{n^2-3}{n-3}$ is a whole number.

7. Consider a table of size 5×10 (the size is not actually important). Can you put some numbers in this table in such a way that the sum in all rows is positive and the sum in all columns is negative?

8. Show that the semi-sum (i.e., half of the sum) of two consecutive odd prime numbers is not prime. (E.g., 13 and 17 are two consecutive odd prime numbers. Their semi-sum $\frac{13+17}{2} = 15$ is not prime.)

9. Take a number which has an odd number of digits (e.g., a number written with 3 digits). Write it next to itself. Show that the resulting number is always divisible by 11. (For example, taking 21356 you would get the number 2135621356 which is divisible by 11.)

10. Show that the sum of any 12 consecutive numbers is not divisible by 4.

11. The numbers n and $n^2 + 2$ are prime. Show that the number $n^3 + 2$ is also prime.