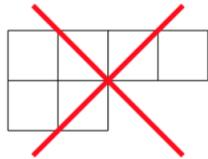


# How to break a number into parts: Young's Diagrams

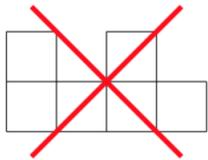
We can represent numbers as a sum of its parts. How can we represent the number six? Let's build a house. Each square will be called an "apartment."

## Rules

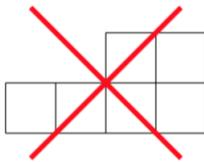
1. An upper floor cannot have more apartments than the lower floor, otherwise the house will crumble.



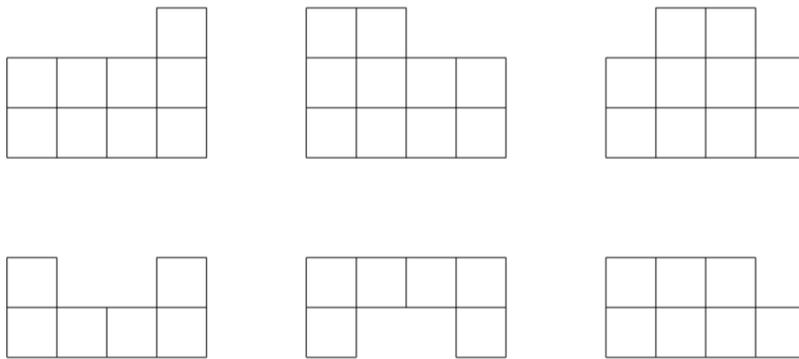
2. There should be no gaps between the apartments (otherwise the house may crumble, too).



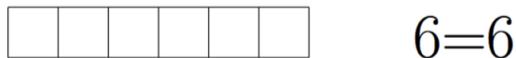
The tallest part of the building must be on its left-hand side (according to the city regulations).



Out of the six houses below, mark out the ones that do not obey our construction rules. What is wrong with them?



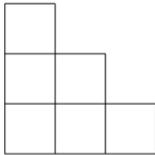
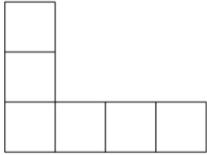
We can call the different ways to build a house by the word “partitions.” Let’s see an example with the number six. First, here comes the one-story house. This is the partition of the number six into one piece.



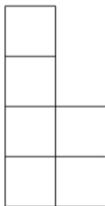
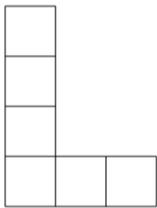
Here come the two-story houses and the corresponding two-parts partitions.



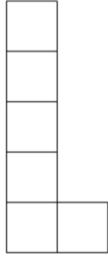
Then the time comes for the three-story houses.



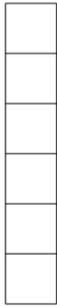
Here come the four-story houses and the corresponding partitions.



There is only one way to build a five-story building (why?).



And finally here comes the six-story tower.



## Problems

1. Construct all the possible houses with 4 apartments by drawing the squares on the grid paper below. Draw only one house per grid. Sum up the number of the apartments on each floor next to the appropriate picture.

- One-story house.



4 =

- Two-story house(s).



4 =



4 =

- Three-story house(s).



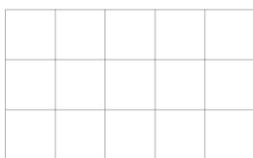
$$4 =$$

- Four-story house(s).



$$4 =$$

2. Find the value of  $2+3$  by drawing a two-story house. There should be 3 apartments on the first floor, and 2 apartments on the second. Don't forget to follow our construction rules! Once the building is finished, count the total number of squares.



$$2 + 3 =$$

- Find two more ways to construct a two-story house to represent  $2+3$ .

3. Build a house that helps you add three and four. Write the corresponding addition problem on the right. (Hint: What should the first floor be?)

4. Find the value of  $7-3$  by drawing a two-story house with seven apartments, 3 of them on the second floor, and by counting the number of apartments on the first floor.

5. Using this approach, find

$$7-4 =$$

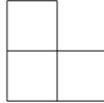
## Even and Odd Numbers with Young's Diagrams

**Definition** A (positive) number is called even if it can be represented by a two-story house with an equal number of apartments on each floor.

1 is obviously not even. It cannot be represented by a two-story house, because we only have one apartment. So, 1 is an odd number. 2 is even.

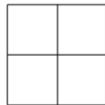


3 is odd.



1. Is there a way to represent 3 as a two-story house with an equal number of apartments on each floor? Why or why not?
2. Below is the house representation for 4. Under it, draw the house for 5.

4 is even.



What is the pattern?

3. Can you draw 9? Prove that it is odd by drawing a picture.

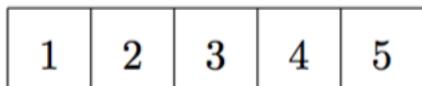
4. Can you draw 12? Prove that it is even by drawing a picture.

5. Find all the partitions of the number 5.

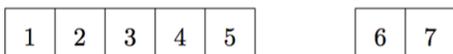
6. Find all the partitions of the number 6 into odd parts. In other words, draw all the possible houses with 6 apartments and with an odd number of apartments on every floor. Hint: first, list all the positive odd integers less than 6.

## Subtraction with Young's Diagrams

We can think of subtraction as building a house and then removing a floor or a few. For example, we can figure out the value of  $7-5$  by drawing a floor with 5 apartments.



Let us draw one more floor, both floors “floating in the air” along a straight line this far, such that the total number of the apartments on both floors equals to seven.

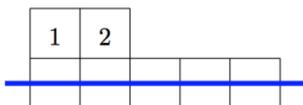


Now, assemble the floors into a proper house.



The above picture shows that we can think of seven as of a whole made of two parts, one of which is five.

To finish the solution, let us mark out the floor with 5 apartments. Counting the remaining apartments gives the result.



$$7 - 5 = 2$$

1. Use the above method to find

$$7-3 =$$

2. Use the above method to find

$$7-4 =$$

3. Solve with the above method.

$$12-4-5-1 =$$

4. Solve with the above method.

$$17-3-3-7-1 =$$