Junior Circle – Games with coins and chessboards

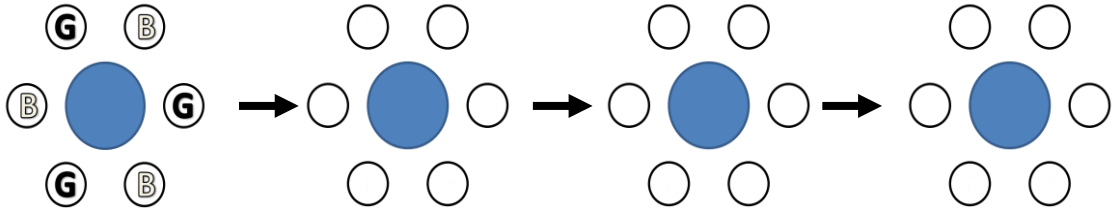
1. a.) There are 4 coins in a row. Let's number them 1 through 4. You are allowed to switch any two coins that have a coin between them. (For example, you can switch the first and third coin). Can you reverse the order of the coins by repeating this operation many times? If yes, show how. If not, explain why not.

   b.) Now what if there are 6 coins in a row numbered 1 through 6. Can you reverse the order of the coins?

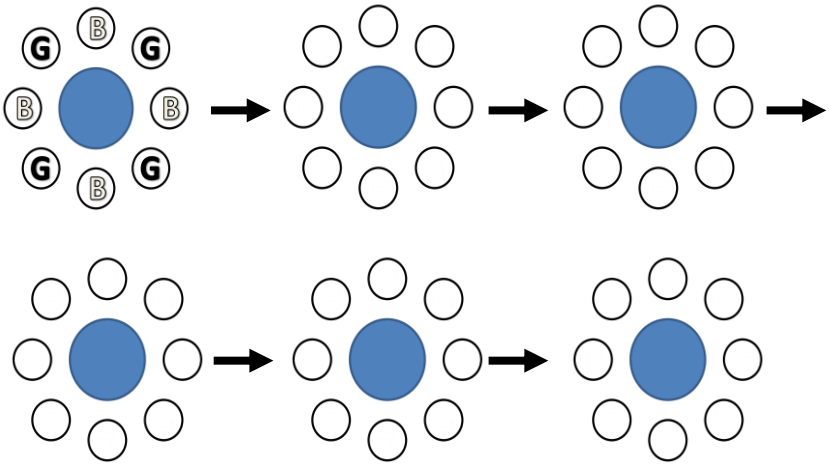
   c.) Now what if there are 5 coins in a row numbered 1 through 5. Can you reverse the order now?

   d.) Looking at the cases above, do you see a pattern? When can the order of the coins be reversed?

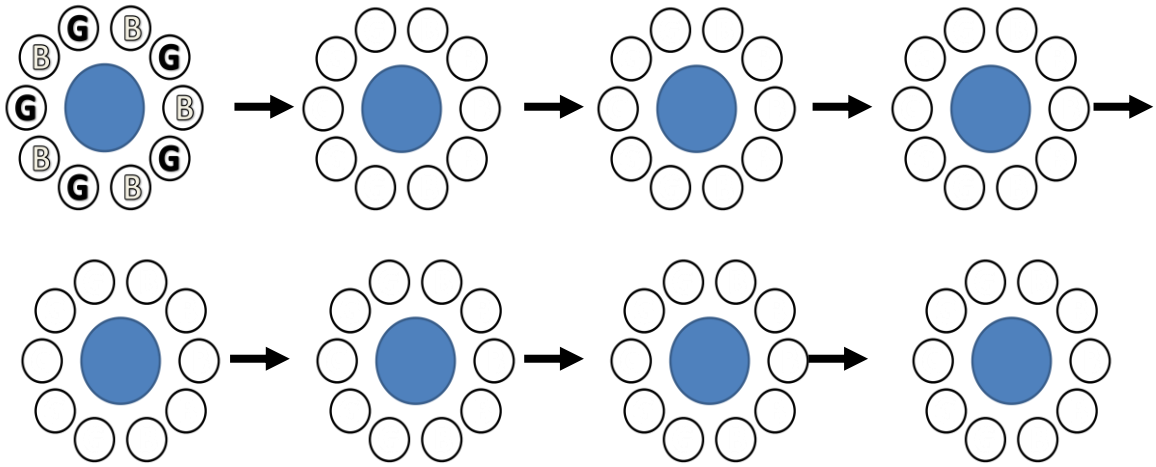
2. 3 boys and 3 girls are sitting around a circular table in alternating order (boy-girl-boy-girl…). They are playing a game where any two kids sitting directly next to each other can swap places. What is the smallest possible number of swaps needed to sit all the girls together and all the boys together?



One more girl and one more boy join the game so that now there are 4 boys and 4 girls and they restart in alternating order. What is the smallest possible number of swaps needed to sit all the girls together and all the boys together?



Another girl and boy pair join the game so that now there are 5 boys and 5 girls and they restart again in alternating order. What is the smallest possible number of swaps needed to sit all the girls together and all the boys together?

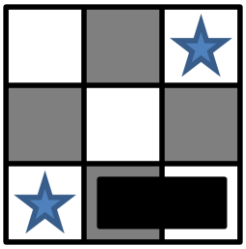


What is the pattern? How many swaps would be needed for 6 girls and 6 boys?

3. Nina has a bag of dominos that she wants to put on chess boards of various sizes. When a domino is placed on the chessboard, it covers two squares that touch each other (not diagonally). Nina wants to place all the dominos on the chess boards so that the lower left and upper right corners are left bare and all other spaces are covered by the dominos. These two corners are marked by stars (★) on the boards below. A domino is drawn like this ▬

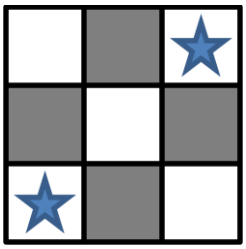
a.) Nina first tries a 2 by 2 sized board. Can a domino be placed on the board so that the lower left and upper right corners are bare?

b.) Nina next tries a 3 by 3 board. Here's a picture of a 3 by 3 board with a domino placed on it:



What can you say about the colors of two squares that a domino covers on a chessboard?

Is there a way to lay down dominos to cover the board so that only the lower left and upper right corners are bare? If yes, show how. If not, explain why not.

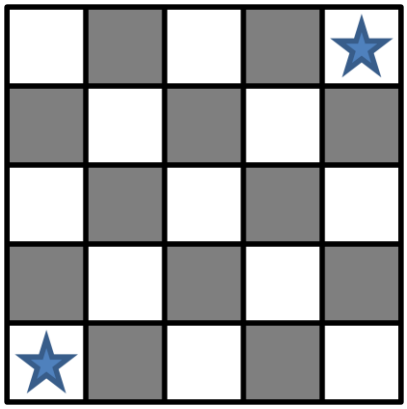


What is the biggest number of dominos you can place on the board above so that the lower left and upper right corners are left out?

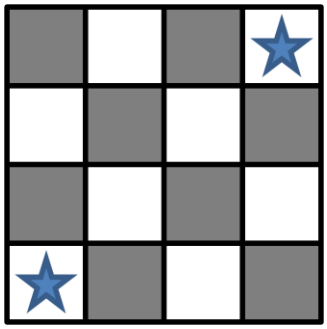
How many chessboard squares are left out?

Why did the 3 by 3 case not work?

Would a 5 by 5 board work?



c.) Knowing that boards with odd numbers of squares don't work, Nina next tries a 4 by 4 board. Is there a way to cover all the squares except the two starred corners?



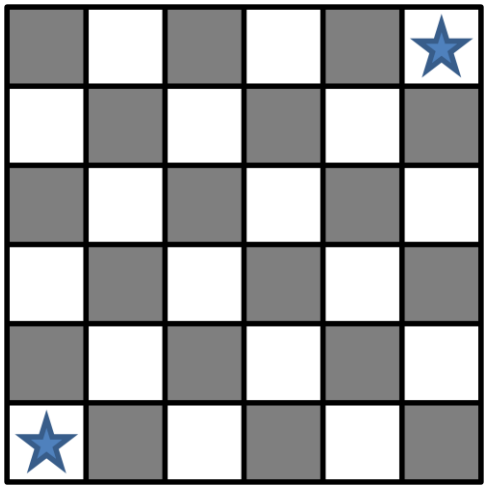
What is the biggest number of dominos you can place on this board so that they do not cover the stars?

How many squares are covered?

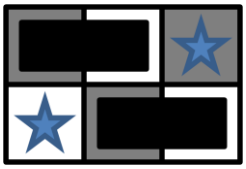
How many squares are left?

Why did the 4 by 4 case not work? (Hint: look at the color of the starred corners)

Would a 6 by 6 board work?



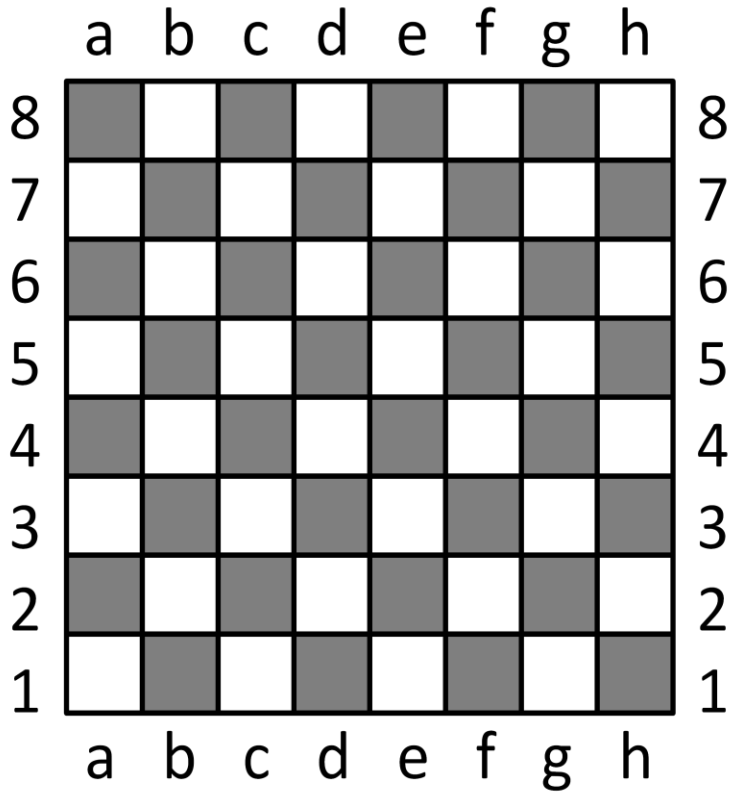
d.) Nina now wants to design a chessboard where she can cover all the spaces except the lower left and upper right corners. She tries a 2 by 3 board and sees that it works.



Why does it work in this case?

Can you come up with any other size boards that work?

4. The rows of a chessboard are numbered 1 through 8 and the columns are lettered a through h:



A Knight stands on a1 on the chessboard above.

a.) From a1, which squares can the Knight jump to in one move?

b.) Explain how the Knight can get to c1.

c.) Explain how the Knight can get to a2.

d.) Explain how the Knight can get to b1.

e.) Can the Knight get from a1 to h8? How many squares did the Knight visit, besides a1?

5. We want to know if the Knight can go from a1 to h8 visiting each square exactly once.
   a.) How many moves are needed to visit each square exactly once?

   b.) How does the color of a square change after a Knight has moved once?

   c.) If a Knight moves an odd number of times, what colored square will he end up on?

   d.) Using this information, can the Knight go from a1 to h8 visiting each square exactly once?

6. Take a look at the expression below. You can put a + or a – sign in every ◯. We want to understand if we can get 0 as the result of this expression by choosing pluses and minuses.

1 ◯ 2 ◯ 3 ◯ 4 ◯ 5 ◯ 6 ◯ 7 ◯ 8 ◯ 9

   The problem is that there are so many choices that it would take us a long time (maybe a whole day!) to try them all. So we will try to understand this without trying all the different possibilities.

a.) What is the value if all of the signs are + ?

b.) Change the first + to -. How did the value change?

c.) Go back to the expression with all pluses. Change the second + to a -. How did the value change?

d.) Change any other plus to a minus. How does the value of the expression change?

e.) Based on your answer, determine whether you can get 0 as the value. Explain why.