Platonic Solids

Christopher Ohrt
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Problem 1. (i) Find all rotational symmetries of a regular n-gon.

(ii) Find all rotational symmetries of the tetrahedron.

Problem 2. (i) How many rotational symmetries does a cube and an octahedron have?

(ii) What do you notice? Can you give a geometric reason why that might be?

(iii) Do any other platonics exhibit such relationships.

(iv) How many rotational symmetries do the dodecahedron and icosahedron have?

Problem 3. Show that the symmetries of the tetrahedron are a subset of the symmetries of a cube. Are these sets identical?

Problem 4. Show that the symmetries of the tetrahedron are a subset of the symmetries of the dodecahedron?

Problem 5. Show that certain edges of the dodecahedron form a cube. Are all the symmetries of the cube also symmetries of the dodecahedron?

Problem 6. Is there some structure or relationships you can impose on the set of rotational symmetries of a platonic or a regular n-gon?

Definition 1. A group G is a set together with a multiplication $(a, b) \mapsto ab \in G$, such that there is an identity object $e \in G$ with $ea = a$ and there is an inverse a^{-1} for all $a \in G$ with $aa^{-1} = a^{-1}a = e$.

Problem 7. (i) What would be easy examples of groups?

(ii) Show that the rotational symmetries discussed above form groups.

(iii) Describe the group of rotational symmetries of a regular n-gons. We call this group $\mathbb{Z}/n\mathbb{Z}$

Problem 8. Does $ab = ba$ always hold for all elements a, b in any group G?

Definition 2. A subset $H \subset G$ of a group is a subgroup if it's a group with the same operation and identity as G.

Problem 9. (i) Show that one can view $\mathbb{Z}/n\mathbb{Z}$ can be viewed as a subgroup of $\mathbb{Z}/nm\mathbb{Z}$. Can you give a geometric interpretation?

(ii) Show that all the set inclusions you proved before are also subgroup-inclusions.