LAMC Intermediate Circle **January 31, 2016**

Oleg Gleizer
prof1140g@math.ucla.edu

Problem 1 *A few decimal numbers constructed using the digit 8 only are written one under another forming a long addition problem. Neither the number of the digits in the numbers, possibly different for some or all the numbers, nor the number of the numbers is known, but we know that the total sum is 1,000.*

$$
\begin{array}{rcccc}
 & * & * & * & 8 \\
 & * & * & * & 8 \\
 & & & & \\
+ & * & * & * & 8 \\
 & & & & \\
 & * & * & * & 8 \\
 & * & * & * & 8 \\
\hline
 & 1 & 0 & 0 & 0
\end{array}
$$

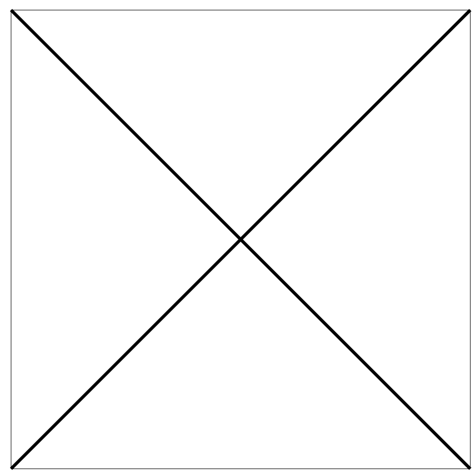
Find the numbers.

Problem 2 *A six-digit number having 1 as its leftmost digit becomes three times greater if we move the digit to the end of the number. What is the number?*

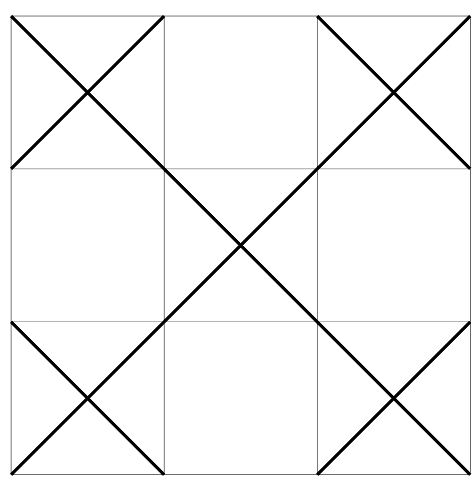
Problem 3 *Given* $\log_b 27 = a$*, find* $\log_{\sqrt{3}} \sqrt[6]{b}$.

Consider the following procedure.

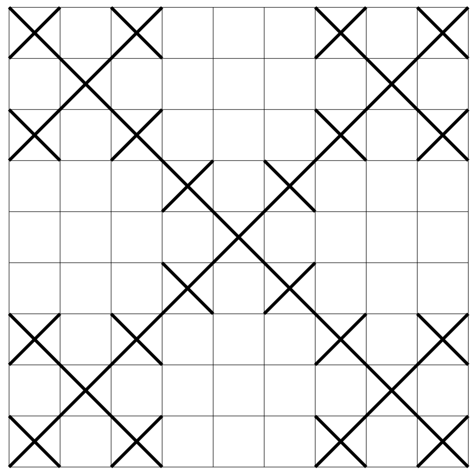
Step 0: take a square of side length a and draw its diagonals.



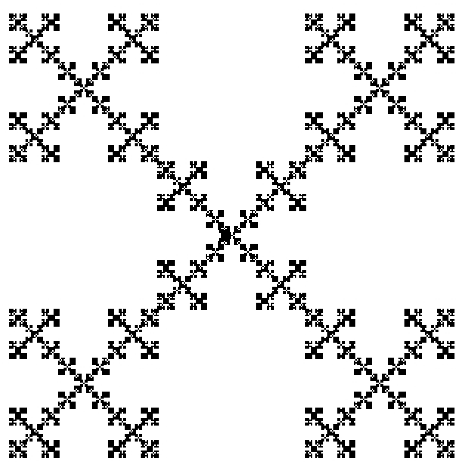
Step 1: divide the square into nine congruent squares. Draw the diagonals of the corner squares and of the central square.



Step 2: repeat the procedure for each of the five squares with marked diagonals.



Proceed to infinity. The following picture of the resulting fractal is drawn using the Python's Turtle module.



Problem 4 *Find the dimension of the fractal.*

Homework Problem 1 *Use Python to write a recursive function sd(n,a) that draws the fractal of order n starting with the diagonals of a square of side length a (sd stands for square's diagonals).*

Problem 5 *Find a decimal expression for the number* $28\frac{3}{17}$.

Problem 6 *Find a rational expression for the number* $7.0\overline{65}$.

A sequence (finite or infinite) $a_0 = a$, $a_1 = aq$, $a_2 = aq^2$, $a_3 = aq^3$, ... , $a_n = aq^n$, ... is called a *geometric sequence*. The number q is called its *ratio*.

Problem 7 *The first term of a geometric sequence is 1, the third term is 4. Find the second term. Is your answer the only one possible?*

Problem 8 *A bacterium dividing one a minute fills a vessel in 30 minutes. How much time would it take for two bacteria to fill the same vessel?*

Problem 9 *A finite geometric sequence with the ratio* $q \neq 0$ *is rewritten in the reversed order, from right to left. Do we get a geometric sequence? If so, what is its ratio?*

Problem 10 *The first term of a geometric sequence is* a, *the third term is* b. *Find the second term.*

Problem 11 *Is it possible that the first 10 terms of a geometric sequence are integers, but all succeeding terms are not?*

Problem 12 *Is it possible that the second term of a geometric sequence is less than its first term and also less than its third term?*

Problem 13 *Is it possible that the numbers 2, 3, and 5 are (not necessarily adjacent) terms of a geometric sequence?*

Problem 14 *Find a formula for the sum of the first* n *terms of a geometric sequence.*

Problem 15 *Find a formula for the sum of all the terms of an infinite geometric sequence with the ratio* $|q| < 1$.

The following is a version of the famous *Zeno's paradox*. Achilles, considered the best ever runner of the Ancient Greece, has a race with a tortoise. The reptile is given a 10 meters head start. Achilles runs 10 times faster than the tortoise. By the time he covers 10 meters, the reptile covers only one. By the time Achilles covers that meter, the tortoise covers 0.1 meter. By the time Achilles covers the 0.1 meter, the reptile covers 0.01 meter, and so on.

Let us call δ the amount of time it takes Achilles to run 10 meters. Then the situation above is described by the following table.

time	0	δ	1.1δ	$1.11\ \delta$	1.111δ	...
d_A	0	10	11	11.1	11.11	...
d_t	10	11	11.1	11.11	11.111	...

It looks like the sluggish animal always stays ahead of the best runner of the Antiquity!

Problem 16 *Use geometric sequences to disprove the above argument.*

Problem 17 *Find the first three members of an infinite geometric sequence with the ratio* $|q| < 1$ *such that the sum of the first five terms of the sequence equals 93/16 and the sum of all the (infinitely many) terms equals 6.*

Problem 18 *Find the decimal rational form of the binary number* 101.01011_2.

Problem 19 *Find the decimal rational form of the binary number* $101.010\overline{11}_2$.

Problem 20 *Find the decimal rational form of the hexadecimal number* $a.bc$.

Problem 21 *Find the decimal rational form of the hexadecimal number* $a.\overline{bc}$.

Problem 22 *Find the decimal rational form of the decimal number* $12.3\overline{45}$.

Problem 23 *Give an example of an octal (base 8) number that is not an octal form of a rational decimal number.*