

Junior Circle Meeting #7: Translations and Rotations on the plane

February 21, 2010

Translations

1. How do we get arrow #2 from arrow #1 below? Describe the process clearly.



2. Now draw a path following the bottom point of arrow #1 to the bottom point of arrow #2. Do the same for the top point of the arrows. What do you notice about the lines you have drawn?
3. What do you notice about the size of arrow #1 compared to the size of arrow #2?

4. What can you say about the directions of arrow #1 and arrow #2?

5. What can you say about the distance traveled by both the top point and the bottom point of arrow #1?

6. Arrow #4 is obtained by changing arrow #3 in such a way that both the top and the bottom points travel the same distance. How is the relation between the arrows #3 and #4 different from the relation between the arrows #1 and #2?



7. Arrow #5 is obtained from arrow #6 by moving both top and bottom points in the same direction. How is the relation between the arrows #5 and #6 different from the relation between the arrows #1 and #2?



8. The process of obtaining arrow #2 from arrow #1 (in problem 1) is an example of **translation**.

What are the two things you can say about the movement of the top and bottom of the arrow when you are performing a translation:

(1)

(2)

9. Are the same statements true for all other points (not just top and bottom) on the arrows? Why or why not?

10. Explain why the change of arrow #3 into arrow #4 is not a translation.

11. Explain why the change of arrow #5 into arrow #6 is not a translation.

12. Consider two translations performed one after the other:

- First, translate arrow #1 into arrow #2;
- Second, translate arrow #2 into arrow #3;



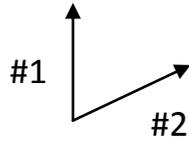
Can you get directly from arrow #1 to arrow #3? Is this a translation?

13. Place an arrow on the map of the United States so that the arrow is on Los Angeles. First, translate the arrow north by 1 cm. Then, translate the arrow west by 1 cm. Then go south by 2 cm. In what direction (relative to Los Angeles) did your arrow move? (You can sketch this below).



Rotations

14. How can we get arrow #2 from arrow #1 below? Describe the process clearly.



15. How can we get arrow #2 from arrow #1 below? Describe the process clearly. (There are at least two different possible answers to this question. Can you find both of them? Which one is similar to your answer in the previous problem?)



16. This process is called a **rotation**. When we rotate an object, we move it in a circular motion around a fixed point, called the **center**. Find the center of rotation which moves arrow #1 into arrow #2.



17. What do you notice about the size of arrow #1 compared to the size of arrow #2?

18. What can you say about the directions of arrow #1 and arrow #2?

19. Draw the path along which the top of the arrow moved.
Draw the path along which the bottom of arrow moved.
Are the distances the same or different?

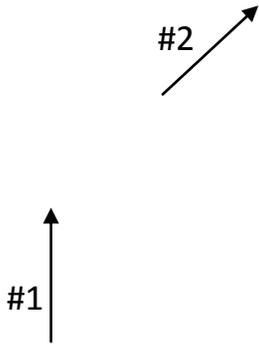


20. What are the similarities of a translation and a rotation?

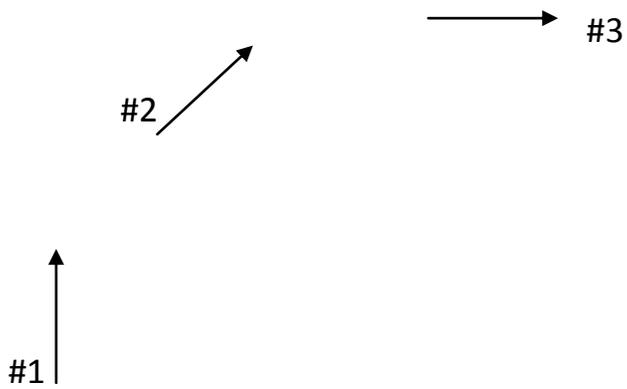
21. What are the differences of a translation and a rotation?

22. Let's say that an arrow moves into itself (so that it actually stays in the same place). Can you say that this is a translation? Can you say that this is a rotation?

23. How do we get arrow #1 from arrow #2 below? Draw the center of rotation.



24. Now we rotate arrow #2 the same angle to get arrow #3.

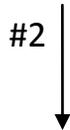


Find the center of rotation from arrow #2 to arrow #3. Is it the same as the center of rotation from arrow #1 to arrow #2?

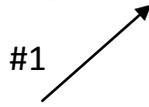
How do we get arrow #3 from arrow #1?

If we keep rotating this arrow, what shape does the bottom point of the arrow trace out?

25. A rotation by half turn can also be thought of as a reflection with respect to the center of rotation. Draw the center of rotation for the arrows below:

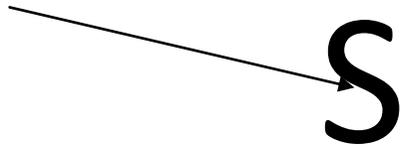


26. Now draw the center of rotation and describe how we can reflect the top and bottom points of arrow #1 to get arrow #2 below.



27. What happens when we rotate the letter S a half turn (180 degrees) around the point below?

Rotate around here



Draw the result below:

28. The letter S has “rotational symmetry”, which means that when we rotate it we can get back the same thing. Are there any other letters in the English alphabet that have this property? Try to name as many as you can!

29.a. Can you get arrow #2 from arrow #1 below? If a rotation was used, draw the center of rotation. If a translation was used, draw the direction of translation. Describe clearly the order of operations.



b. Is there another way to get arrow #2 from arrow #1?

30. Below we have an arrow that is fixed at its bottom point. We rotate it with constant speed in the clockwise direction, and at the same time, the length of the arrow increases. Draw the image that the top point of our arrow traces out as it rotates and lengthens. What is this type of shape called?

