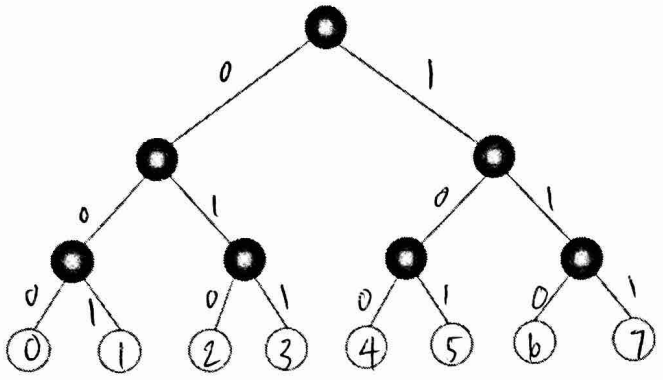
Meeting 5: Binary Part 3

Binary Tree



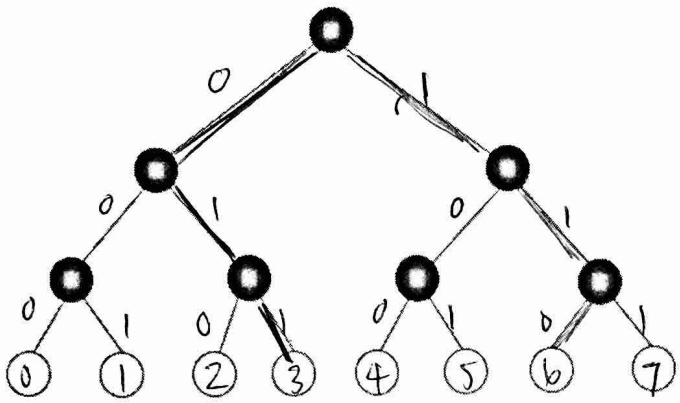
1. Label the vertices of the bottom row by numbers 0 through 7 (going from left to right). You may put numbers right inside of the circles).

2. Play the game "Guess my number" (with numbers from 0 to 7). The goal is to be able to guess the number from 3 attempts (or less, if you are lucky). Can you think of how the edges can help you to formulate the strategy? What is the best way to play the game?

   strategy: ask series of yes/no questions
   (EX. for the 1st branch, do you go 0 or 1? etc...)

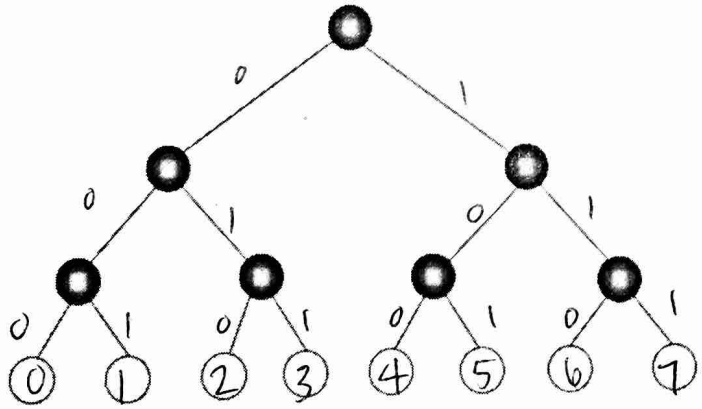
3. Label all the edges pointing to the left by 0. Label all the edges pointing to the right by 1.

4. For each number on the bottom, there is exactly one path (route) from the top circle down to this number.



- Mark the path from the top circle to the number 3;
- Mark with a different color the path from the top circle to the number 6.

5. For each number, the path connecting the top circle with this number gives you a string of 0s and 1s.



(a) Going from top to bottom, write down the string of 0s and 1s along the path going from the top circle to the number 2.

010 → [1][0], which is 2 in binary notation

(b) Write down the string of 0s and 1s along the path from the top circle to the number 6.

(c) Do you recognize your answers in (a) and (b)? (You can ignore the 0s in the beginning of the string). What do these answers represent?

They are binary # representations of the digits on the bottom.

6. How many questions do you have to ask the guess the number in the "Guess my number" game with numbers ranging from

(a) from 0 through 7?

3

(b) from 0 through 15?

4

(c) from 0 through 31?

5

A card trick

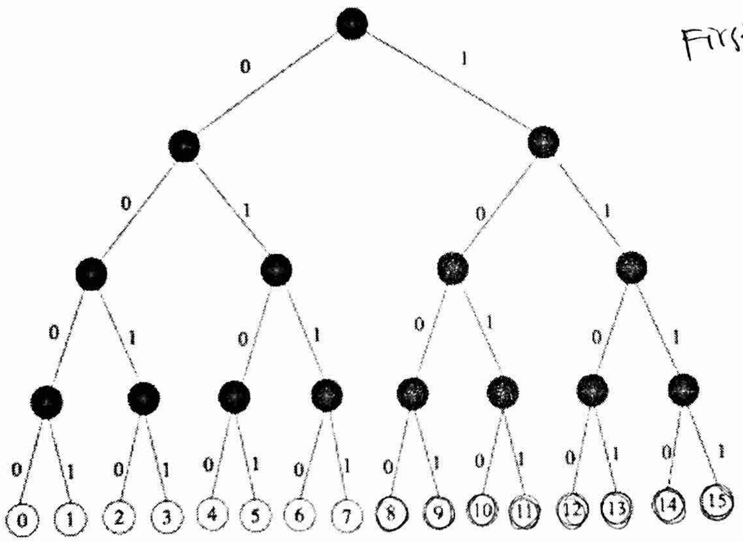
I have 4 cards with numbers 1 through 15 written on them. (Note that most of the numbers appear on several cards). Here are the cards:

$$I = \begin{pmatrix} 8 & 9 & 10 & 11 \\ 12 & 13 & 14 & 15 \end{pmatrix} \quad II = \begin{pmatrix} 4 & 5 & 6 & 7 \\ 12 & 13 & 14 & 15 \end{pmatrix} \quad III = \begin{pmatrix} 2 & 3 & 6 & 7 \\ 10 & 11 & 14 & 15 \end{pmatrix} \quad IV = \begin{pmatrix} 1 & 3 & 5 & 7 \\ 9 & 11 & 13 & 15 \end{pmatrix}$$

All these numbers can be written in binary notation using 4 digits (for some numbers, the first digit(s) can be 0s).

1. On the binary tree below, mark the numbers appearing on Card 1. What do they have in common?

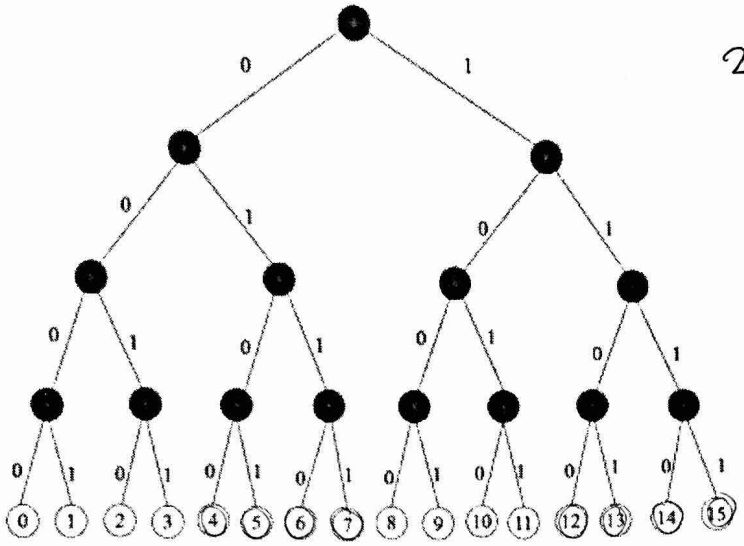
First binary # is 1



2. What is the first binary digit of all the numbers appearing on Card 1?

1

3. On the binary tree below mark the numbers appearing on Card 2. What do they have in common?

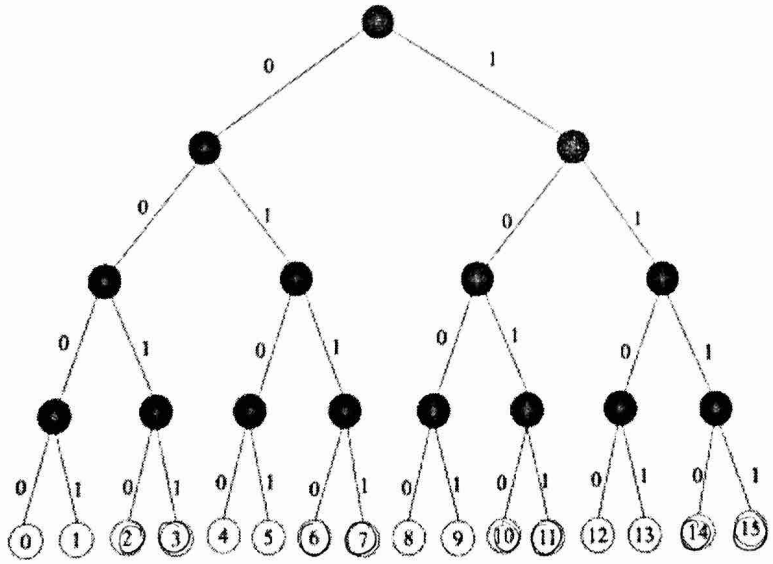


2nd binary digit is 1

4. What is the second binary digit of all the numbers appearing on Card 2?

Second binary digit is 1

5. On the binary tree below mark the numbers appearing on Card 3. What do they have in common?

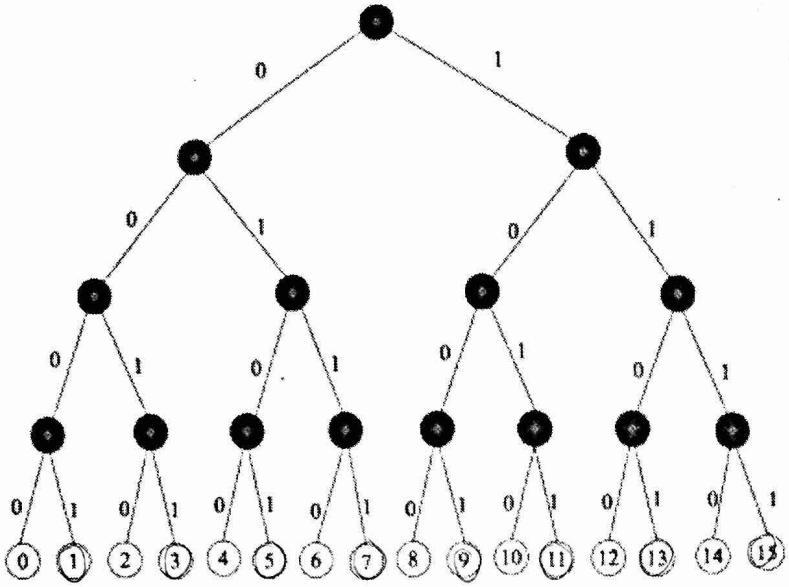


3rd binary digit is 1

6. What is the third binary digit of all numbers appearing on Card 3?

1

7. On the binary tree below mark the numbers appearing on Card 4. What do they have in common?



Odd #'s
or
4th binary digit is 1

8. What is the last binary digit of all numbers appearing on Card 4?

4th binary digit is 1

9. If I tell you that the number I am thinking of has "1" as its first binary digit, can you tell me which card I am holding?

yes - it is the 1st card.

10. How can you tell what number you have given the numbers of the cards it is written on (think about the strategy used in "Guess my number")?

Ex. given card I, know all of the 1st binary digit is 1. Then ask yes/no answers until you get the # (i.e. is the 2nd binary digit 0? etc...)

→ Use strategy from "guess my #" to narrow down possibilities.

Addition

32 16 8 4 2 1

Recall addition rules
* 0 + 0 = 0
* 1 + 0 = 0
* 1 + 1 = 10

Do the following addition problems.

1. $5+7=$

   (a) Convert into binary notation

   101
   + 111

   (b) Add in binary notation.

   101
   + 111
   1100

   remember to carry!

   (c) Convert the answer into decimal notation.

   1100 = 12

   (d) Check the addition in decimal notation.

   5+7=12 ✓

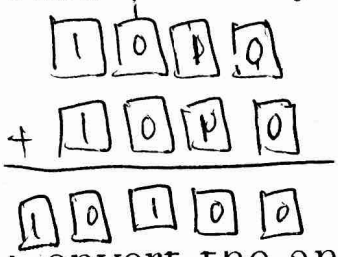
2. $10+10=$

   (a) Convert into binary notation.

   1010 + 1010

32 16 8 4 2 1

(b) Add in binary notation.



(c) Convert the answer into decimal notation.

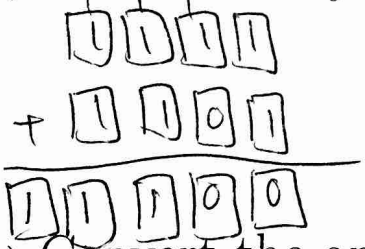
(d) Check the addition in decimal notation.

10+10=20 ✓

3. $15 + 13 =$

(a) Convert into binary notation.

(b) Add in binary notation.



(c) Convert the answer into decimal notation.

(d) Check the addition in decimal notation.

15+13 =28 ✓

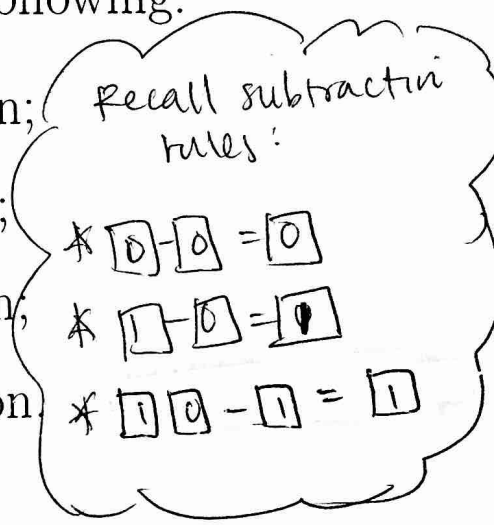
My

32 16 8 4 2 1

Subtraction

For the following addition problems, do the following:

- Convert the numbers into binary notation;
- Subtract the numbers in binary notation;
- Convert the answer into decimal notation;
- Check the subtraction in decimal notation.



1. $7 - 5 =$

   (a) Convert into binary notation.

   111 − 101

   (b) ~~Add~~ Subtract in binary notation.

   111
   − 101

   10

   (c) Convert the answer into decimal notation.

   10 = 2

   (d) Check the addition in decimal notation.

   7 − 5 = 2 ✓

32 16 8 4 2 1

2. $6 - 5 =$

(a) Convert into binary notation.

1 1 0 − 1 0 1

(b) ~~Add~~ Subtract in binary notation.

1 1 0
− 1 0 1

1

(c) Convert the answer into decimal notation.

1 = 1

(d) Check the addition in decimal notation.

6 − 5 = 1 ✓

3. $13 - 7 =$

(a) Convert into binary notation.

1 1 0 1 − 1 1 1

(b) ~~Add~~ Subtract in binary notation.

1 1 0 1
− 1 1 1
---
1 1 0

32 16 8 4 2 1

(c) Convert the answer into decimal notation.

1 1 0 = 6

(d) Check the addition in decimal notation.

13 - 7 = 6 ✓

4. What is the *largest* number that can be written in binary notation using two 1s and a 0?

1 1 0

Convert this number into decimal notation:

6

5. What is the *smallest* number that can be written in binary notation using two 1s and a 0?

1 0 1 * do not put 0 at the very front (against binary notation)

Convert this number into decimal notation:

5

6. What is the *largest* number that can be written in binary notation using two 1s and two 0s?

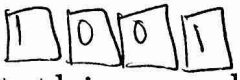
1 1 0 0

Convert this number into decimal notation:

12

32

7. What is the *smallest* number that can be written in binary notation using two 1s and two 0s?



Convert this number into decimal notation:

9

8. For the two numbers written in binary notation below, decide which one is even and which one is odd:

$$1001***10$$ even

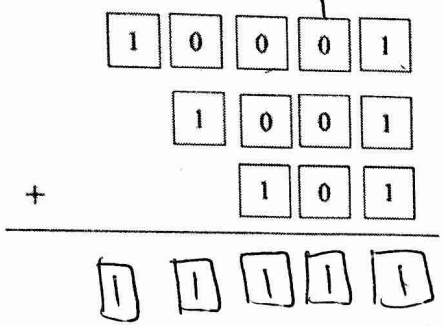
$$1001***01$$ odd (1 @ the end in 1's place)

Here $***$ stand for any binary digits.

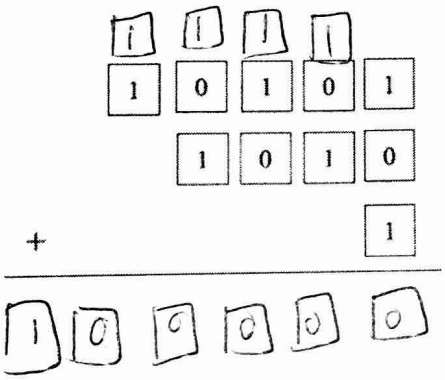
Addition and Subtraction Challenge Problems

32 16 8 4 2 1

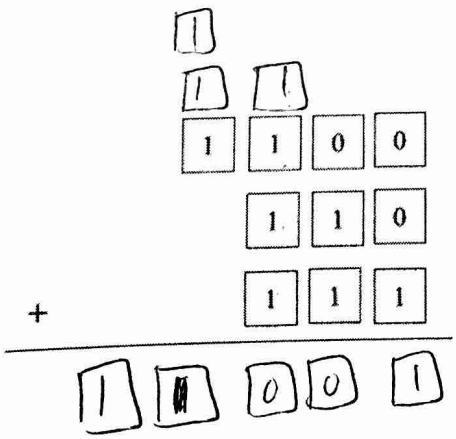
Solve the following addition and subtraction problems. Remember to box your digits.



check:
17
+ 9
5
31 ✓

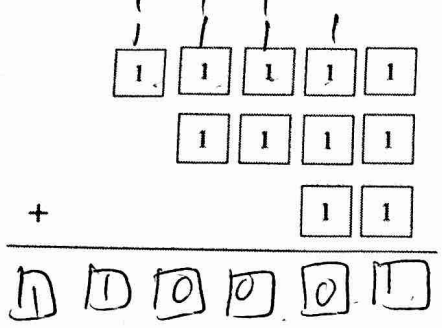


check:
21
10
+ 1
32 ✓

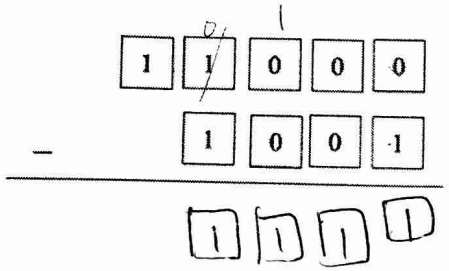


check:
12
6
+ 7
25 ✓

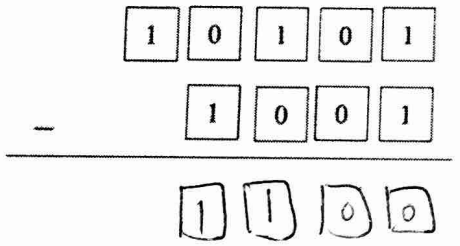
32 16 8 4 2 1



check:
31
15
+ 3
49 ✓



check:
24
− 9
15 ✓



check:
21
− 9
12 ✓