Platonic Solids

Christopher Ohrt
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Definition 1. A platonic solid is a three dimensional polytope where all faces are isometric regular polygons such that the same number of edges meet at each vertex under the same angles.

Problem 1. Which platonic solids do you know? Do you think that there are more? Why?

Problem 2. We can turn any polytope into a graph by drawing a vertex for every vertex of the polytope and connect them by edges according to the edges in the polytope. If we consider the (infinitely big) outside area of a graph also an area closed off by the graph then all the seperated areas of the graph correspond to the faces of the polytope. Draw the graphs for all the platonic solids you found in problem 1.

Problem 3. Let V be the number of vertices, E the number of edges and F the number of faces (including the outside face) of a graph. Euler introduced the following formula:

$$V - E + F = 2.$$

(i) Check Euler's formula on the platonic solids you found earlier.

(ii) Prove Euler's formula using induction (start with the graph with only one vertex).

Problem 4. Now assume you are given a platonic solid the faces of which are regular n-gons and with k edges meeting at each vertex.

(i) Give formulas relating the number of faces F, edges E, and vertices V of this solid.

(ii) Can you now use Euler's formula to find all platonic solids and prove that there are not any other ones.

Problem 5. Draw nets for all platonic solids and identify the edges that will be glued together. *Bonus:* Cut out, fold, and glue your platonics.

Problem 6. Consider the icosahedron (20-faced platonic) standing on one of its vertices (so that there also is a sharp tip on top). Now "dent in" the top and the bottom tip such that they both meet in one point in the center of the solid.

(i) Sketch the resulting solid.

(ii) Check Euler's formula for this solid.

(iii) What went wrong?

Problem 7. Find more solids for which Euler's formula does not hold.

Problem 8. Can you give a proof why a sphere cannot be deformed into a donut "without ripping".