

More Adventures in Binary Land

Decimal Notation:

1. What is the value of digit 8 in the number 938?

8

2. What is the value of digit 8 in the number 581?

80

3. What is the value of digit 8 in the number 824?

800

4. Let's look at the number 3456

- (a) What is the value of the digit 3?

3000

- (b) What is the value of the digit 5?

50

Binary Notation:

1. Put the numbers 7, 15, 20, 29, 36, 43 and 50 into binary notation by filling out the table below:

Number	32	16	8	4	2	1
7	0	0	0	1	1	1
15	0	0	1	1	1	1
20	0	1	0	1	0	0
29	0	1	1	1	0	1
36	1	0	0	1	0	0
43	1	0	1	0	1	1
50	1	1	0	0	1	0

- (a) What is the value of the rightmost digit 1 in the binary notation of the number 7?

1

- (b) What is the value of the rightmost digit 1 in the binary notation of the number 20?

4

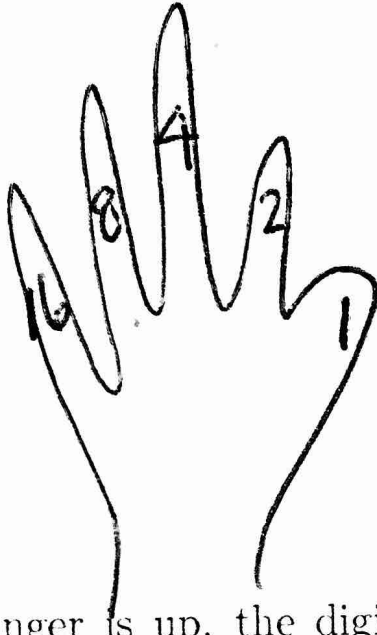
- (c) What is the value of the rightmost digit 1 in the binary notation of the number 36?

32

<---error in the answer key: should be 4

Counting with your fingers!

Each finger represents a binary digit of a number;



If a finger is up, the digit is 1; if a finger is down, the digit is 0. (For the best results, put your hand flat against the table to keep the finger position.)

Have your instructor label your fingers for you.

Write the following numbers in binary notation, with the help of your fingers.

1. $5 =$

1	0	1
---	---	---

2. $9 =$

1	0	0	1
---	---	---	---

3. $17 =$

1	0	0	0	1
---	---	---	---	---

4. $28 =$

1	1	1	0	0
---	---	---	---	---

5. What is the highest number your can count to on the fingers of one hand in binary notation?

$$16 + 8 + 4 + 2 + 1 = 31$$

6. For each of the binary numbers, implement them on your hand (press the hand on the table as you are doing this to make it easier). Then add together the values of all the digits to get the decimal number:

(a) $\boxed{1} \boxed{1} = 2 + 1 = 3$

(b) $\boxed{1} \boxed{0} \boxed{0} \boxed{0} \boxed{1} = 16 + 1 = 17$

(c) $\boxed{1} \boxed{0} \boxed{1} \boxed{1} \boxed{1} = 16 + 4 + 2 + 1 = 23$

(d) $\boxed{1} \boxed{1} \boxed{0} \boxed{1} \boxed{1} = 16 + 8 + 2 + 1 = 27$

(e) $\boxed{1} \boxed{1} \boxed{0} \boxed{1} \boxed{0} \boxed{1} \boxed{0} = 64 + 32 + 8 + 2 = 106$

Explain the following addition problem in decimal notation:

$$\begin{array}{r} \\ + \\ \hline 1 \end{array}$$

Now look at a similar problem in binary notation.

$$\begin{array}{r} \\ + \\ \hline 1 \end{array}$$

Here are the addition facts that you need when adding numbers in binary notation:

* $\boxed{0} + \boxed{0} = \boxed{0}$

* $\boxed{0} + \boxed{1} = \boxed{1}$

* $\boxed{1} + \boxed{1} = \boxed{1} \boxed{0}$

↑ Typo: should be 1

4. Explain these addition facts by converting numbers to decimal notation.

#1 $\boxed{0} + \boxed{0} = \boxed{0}$ ✓

#2 $\boxed{0} + \boxed{1} = \boxed{1}$ ✓

#3 $\boxed{1} + \boxed{1} = \boxed{1} \boxed{0}$ ✓

$1 + 1 = 2$

5. Explain which of these addition facts will lead to carrying when you adding bigger numbers. (Hint: think about why you have to carry in decimal notation!)

$1 + 1 = 10$ will lead to carrying.

6. Solve the following addition problems (Be careful as carrying may be involved). Check your work by solving the problem in decimal notation on the side.

1.

1	0	1	0
+	1	0	1
<hr/>			
1	1	1	1

10
+ 5
<hr/>
15 ✓

1	0	1	1
+	1	0	0
<hr/>			
1	1	1	1

11
+ 4
<hr/>
15 ✓

	1			
	1	0	1	1
+	1	0	1	0
<hr/>				
1	0	1	0	1

11
+ 10
<hr/>
21 ✓

* Needs to be carried

$$\begin{array}{r}
 \boxed{1} \boxed{1} \\
 \boxed{1} \boxed{1} \boxed{1} \boxed{1} \\
 + \boxed{1} \boxed{1} \boxed{1} \boxed{0} \\
 \hline
 \boxed{1} \boxed{1} \boxed{1} \boxed{0} \boxed{1}
 \end{array}$$

$$\begin{array}{r}
 15 \\
 + 14 \\
 \hline
 29
 \end{array}$$

✓

* Carrying
is
necessary

Now, let's try subtracting. Here again are some standard subtraction facts we know.

$$\boxed{1} - \boxed{1} = \boxed{0}$$

makes sense

$$\rightarrow 1 - 1 = 0 \checkmark$$

$$\boxed{1} - \boxed{0} = \boxed{1}$$

since
the equivalent

$$\rightarrow 1 - 0 = 1 \checkmark$$

$$\boxed{1} \boxed{0} - \boxed{1} = \boxed{1}$$

is

$$\rightarrow 2 - 1 = 1 \checkmark$$

7. Subtract the following binary numbers:

$$\begin{array}{r}
 \boxed{1} \boxed{1} \boxed{1} \boxed{1} \\
 - \boxed{1} \boxed{0} \boxed{0} \boxed{1} \\
 \hline
 \boxed{0} \boxed{1} \boxed{1} \boxed{0}
 \end{array}$$

$$\begin{array}{r}
 15 \\
 - 9 \\
 \hline
 6
 \end{array}$$

✓

$$\begin{array}{r}
 \begin{array}{|c|c|c|c|} \hline 1 & 0 & 1 & 1 \\ \hline \end{array} \\
 - \quad \begin{array}{|c|c|} \hline 1 & 1 \\ \hline \end{array} \\
 \hline
 \begin{array}{|c|c|c|c|} \hline 1 & 0 & 0 & 0 \\ \hline \end{array}
 \end{array}
 \quad
 \begin{array}{r}
 11 \\
 - 3 \\
 \hline
 8 \quad \checkmark
 \end{array}$$

↩ borrow

$$\begin{array}{r}
 \begin{array}{|c|c|c|c|} \hline 1 & 1 & 0 & 1 \\ \hline \end{array} \\
 - \quad \begin{array}{|c|c|} \hline 1 & 1 \\ \hline \end{array} \\
 \hline
 \begin{array}{|c|c|c|c|} \hline 1 & 0 & 1 & 0 \\ \hline \end{array}
 \end{array}
 \quad
 \begin{array}{r}
 13 \\
 - 3 \\
 \hline
 10 \quad \checkmark
 \end{array}$$

$$\begin{array}{r}
 \begin{array}{|c|c|c|c|} \hline 1 & 0 & 0 & 1 \\ \hline \end{array} \\
 - \quad \begin{array}{|c|c|c|} \hline 1 & 1 & 0 \\ \hline \end{array} \\
 \hline
 \begin{array}{|c|c|} \hline 1 & 1 \\ \hline \end{array}
 \end{array}
 \quad
 \begin{array}{r}
 9 \\
 - 6 \\
 \hline
 3 \quad \checkmark
 \end{array}$$

Compute the sum $1 + 1 + 2 + 4 + 8 + 16 + 32$ like this: first, convert all numbers to binary. Then carry out the addition. Last, convert to a decimal number. (It is not a typo, there are two 1s in this sum!).

0	0	0	0	0	1
<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>
<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>
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<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>

$$\boxed{1}\boxed{1}\boxed{0}\boxed{0}\boxed{0}\boxed{0}\boxed{0} = 64 \quad \checkmark \quad \text{since}$$

$$1 + 1 + 2 + 4 + 8 + 16 + 32 = 64$$