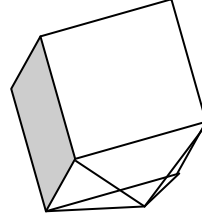


Math for Christmas and Hanukkah

1. Below is a picture of a dreidel and next to it a simplified version of it.



- a. Is a dreidel a polyhedron? Why or why not?

- b. Can you make the dreidel out of two polyhedra that we studied last time?

- c. Count the number of edges, faces, and vertices.

- d. Does Euler's formula work on the dreidel?

2. Now we will answer questions about the song “12 Days of Christmas”:

a. How many gifts (in total) did we get on the first day of Christmas?

b. How many gifts (in total) did we get on the second day of Christmas?

c. How many gifts (in total) did we get on the third day of Christmas?

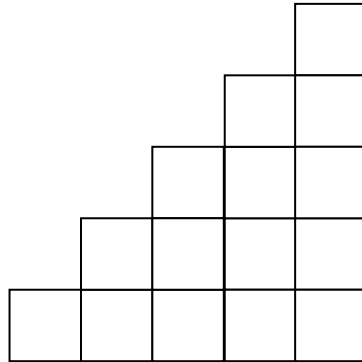
d. Suppose I know the number of gifts I got on the seventh day. How can I get the number of gifts I got on the eighth day from it?

- e. Fill in the table below to count how many gifts we got on each of the twelve days. Use the results from the previous day to compute the next day:

Day of Christmas	Number of gifts received
1	1
2	1+2
3	1+2+3
4	
5	
6	
7	
8	
9	
10	
11	
12	

- f. On each day of Christmas, what is the pattern for how many gifts we received? Use your table above.

g. Let's say we want to know the sum $1+2+3+4+5$ without actually adding all the numbers. We can draw a "staircase" to represent this sum, where the number of squares is $1+2+3+4+5$:



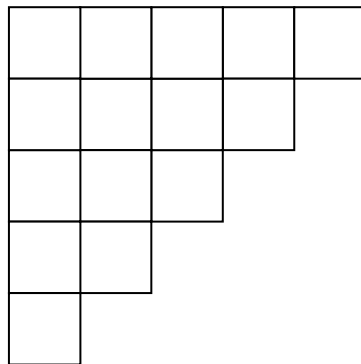
Squares above:

1 2 3 4 5

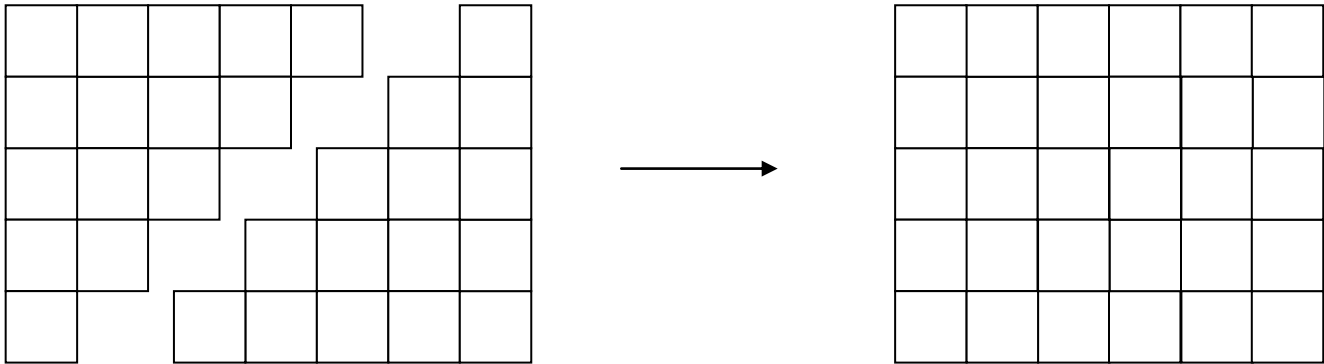
We also draw an upside-down staircase with the same number of squares, $1+2+3+4+5$:

Squares below:

5 4 3 2 1



We now paste the two staircases together to form a rectangle.



Draw a jagged line separating the rectangle into the two staircases.

What are the dimensions (length of sides) of our rectangle?

What is the area (size) of our rectangle?

- h. How do we get the size of our staircase from the size of our rectangle?

- i. Using your answers in parts (g) and (h), write down an expression for the sum $1+2+3+4+5$.

- j. We can also find the sum $1+2+3+4+5$ in the following way. We write the numbers 1 through 5 below and then write the reverse order and add each column. Fill in the table below.

Numbers	1	2	3	4	5
Reverse	5	4	3	2	1
Sum of numbers above					

Do we get the same sum each time?

- k. Now if we add each of these sums, what do we get? You can leave your answer as a multiplication of two numbers.
- l. How do we get the sum $1+2+3+4+5$ from parts (j) and (k)? Do we get the same formula as in part (i)?

m. What will be the formula for $1+2+3+4+5+6$?

n. Bonus question: Can you find the sum:

$$1+2+3+\dots+98+99+100$$

using the same method?

3. Now we want to know how many of each type of gift we got.
 - a. After all 12 days of Christmas, how many partridges in pear trees did we get, in total? Why?

b. How many turtle doves did we get in total?

c. How many French hens did we get in total?

d. Fill in the table below to count how many of each type of gift we got:

Gift	Number received
Partridges in pear trees	12*1
Turtle doves	11*2
French hens	10*3
Calling birds	
Golden rings	
Geese	
Swans	
Maids	
Ladies	
Lords	
Pipers	
Drummers	

e. What is the pattern in the table above?

f. Looking at your table above, which pairs of gifts did we receive the same amount of?

4. In an ancient language, the small numbers are written as follows:

1 = Fa

2 = Fa - La

3 = Fa - Fa

5 = Fa - La - Fa

7 = Fa - Fa - Fa - La

a.) How do you write 0 in this language ?

b.) What numbers do the following words correspond to?

Fa - La - La

Fa - Fa - La - Fa

Fa - La - La - Fa

c.) How can you write 8,10,12,16 in this language ?

5. We wanted to tell you the date of our next Junior Circle meeting, but someone has changed the letters and numbers of the date:

Kbovbsz 21, 3121

a.) Can you figure out the rule that he/she used to change the date?

b.) When will the next meeting be?

c.) What is today's date according to this rule?

On the first day of Christmas,
my true love sent to me
A partridge in a pear tree.

On the second day of Christmas,
my true love sent to me
Two turtle doves,
And a partridge in a pear tree.

On the third day of Christmas,
my true love sent to me
Three French hens,
Two turtle doves,
And a partridge in a pear tree.

On the fourth day of Christmas,
my true love sent to me
Four calling birds,
Three French hens,
Two turtle doves,
And a partridge in a pear tree.

On the fifth day of Christmas,
my true love sent to me
Five golden rings,
Four calling birds,
Three French hens,
Two turtle doves,
And a partridge in a pear tree.

On the sixth day of Christmas,
my true love sent to me
Six geese a-laying,
Five golden rings,
Four calling birds,
Three French hens,
Two turtle doves,
And a partridge in a pear tree.

On the seventh day of Christmas,
my true love sent to me
Seven swans a-swimming,
Six geese a-laying,
Five golden rings,
Four calling birds,
Three French hens,
Two turtle doves,
And a partridge in a pear tree.

On the eighth day of Christmas,
my true love sent to me
Eight maids a-milking,
Seven swans a-swimming,
Six geese a-laying,
Five golden rings,
Four calling birds,
Three French hens,

Two turtle doves,
And a partridge in a pear tree.

On the ninth day of Christmas,
my true love sent to me
Nine ladies dancing,
Eight maids a-milking,
Seven swans a-swimming,
Six geese a-laying,
Five golden rings,
Four calling birds,
Three French hens,
Two turtle doves,
And a partridge in a pear tree.

On the tenth day of Christmas,
my true love sent to me
Ten lords a-leaping,
Nine ladies dancing,
Eight maids a-milking,
Seven swans a-swimming,
Six geese a-laying,
Five golden rings,
Four calling birds,
Three French hens,
Two turtle doves,
And a partridge in a pear tree.

On the eleventh day of Christmas,
my true love sent to me
Eleven pipers piping,
Ten lords a-leaping,
Nine ladies dancing,
Eight maids a-milking,
Seven swans a-swimming,
Six geese a-laying,
Five golden rings,
Four calling birds,
Three French hens,
Two turtle doves,
And a partridge in a pear tree.

On the twelfth day of Christmas,
my true love sent to me
Twelve drummers drumming,
Eleven pipers piping,
Ten lords a-leaping,
Nine ladies dancing,
Eight maids a-milking,
Seven swans a-swimming,
Six geese a-laying,
Five golden rings,
Four calling birds,
Three French hens,
Two turtle doves,
And a partridge in a pear tree!