

# The Fibonacci Sequence

January 17, 2016



Rules:

1. Start with a pair of new rabbits born in December.
  2. When the rabbits are one month old, they are considered teenagers. Teenagers cannot have children.
  3. When the rabbits are two months old, they are considered adults. A pair of rabbits will give birth to a pair of newborn rabbits the same month they become adults.
  4. A pair of adult rabbits gives birth to one pair of newborn rabbits each month.
  5. Rabbits never die. Therefore, once a rabbit becomes an adult, it remains an adult forever.
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1. If a pair of rabbits is born in December, which month will their first pair of children be born in?
2. Suppose there is 1 pair of newborn rabbits in December (Month 0). (a) How many pairs of each type of rabbit (newborn, teenager, and adult) will there be in Month 1?
  - i. Adult:
  - ii. Teenager:
  - iii. Newborn:

b) What about Month 2?

i. Adult:

ii. Teenager:

iii. Newborn:

3. Suppose there are 5 pairs of newborn rabbits in a given month.

(a) How many pairs of adults must there be in that same month?

(b) How many pairs of teenagers will there be in the next month?

4. Suppose there are 3 pairs of newborn rabbits, 1 pair of teenage rabbits, and 3 pairs of adult rabbits in a given month.

(a) How many pairs of each type of rabbit will there be in the next month?

i. Adult:

ii. Teenager:

iii. Newborn:

(b) What about the month after?

i. Adult:

ii. Teenager:

iii. Newborn:

5. Let's figure out what Fibonacci's sequence is. Starting with one pair of newborn rabbits in Month 0, use the rules to fill in the chart below:

|          | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|----------|---|---|---|---|---|---|---|---|---|---|----|
| Newborn  | 1 |   |   |   |   |   |   |   |   |   |    |
| Teenager | 0 |   |   |   |   |   |   |   |   |   |    |
| Adult    | 0 |   |   |   |   |   |   |   |   |   |    |
| Total    | 1 |   |   |   |   |   |   |   |   |   |    |

6. The sequence of numbers that represent the total number of pairs of rabbits is known as the Fibonacci sequence!
- (a) How is the total number of pairs of rabbits in Month 2 related to the total number of pairs in Month 0 and Month 1?
- (b) How is the total number of pairs in Month 5 related to the total number of pairs in Month 3 and Month 4?
- (c) How is the total number of pairs in Month N related to the total number of pairs in the previous months?

7. We are going to look at how the rabbits change through a period of 3 months. Fill out the table at the bottom of the page using the following directions:
- Draw a point in the center of the box that corresponds to teenagers from 2 months ago. Draw an arrow that starts from this point and ends in the center of the box that corresponds to adults from last month. An example has already been done with the newborns from 2 months ago.
  - Remember that once a rabbit is an adult, it stays an adult forever. Using an arrow, show where the adults from 2 months ago ended up being last month.
  - Draw a curved arrow that starts from adults from last month and ends in newborns from last month. This curved line should not intersect any lines you have already drawn. (d) Draw arrows that show where each type of rabbit from last month end up being in the current month.
  - Draw a curved arrow that starts from adults in the current month and ends in newborns in the current month. This curved line should not intersect any of the lines you have already drawn.
  - Using your purple colored pencil, draw over the arrows that connect all the rabbits from last month to the current month.
  - Using your green colored pencil, draw over the curved arrow that shows that newborns in the current month come from adults in the current month. (h) Using your green colored pencil, draw over the arrows that show how all the rabbits from 2 months ago end up being adult rabbits in the current month.

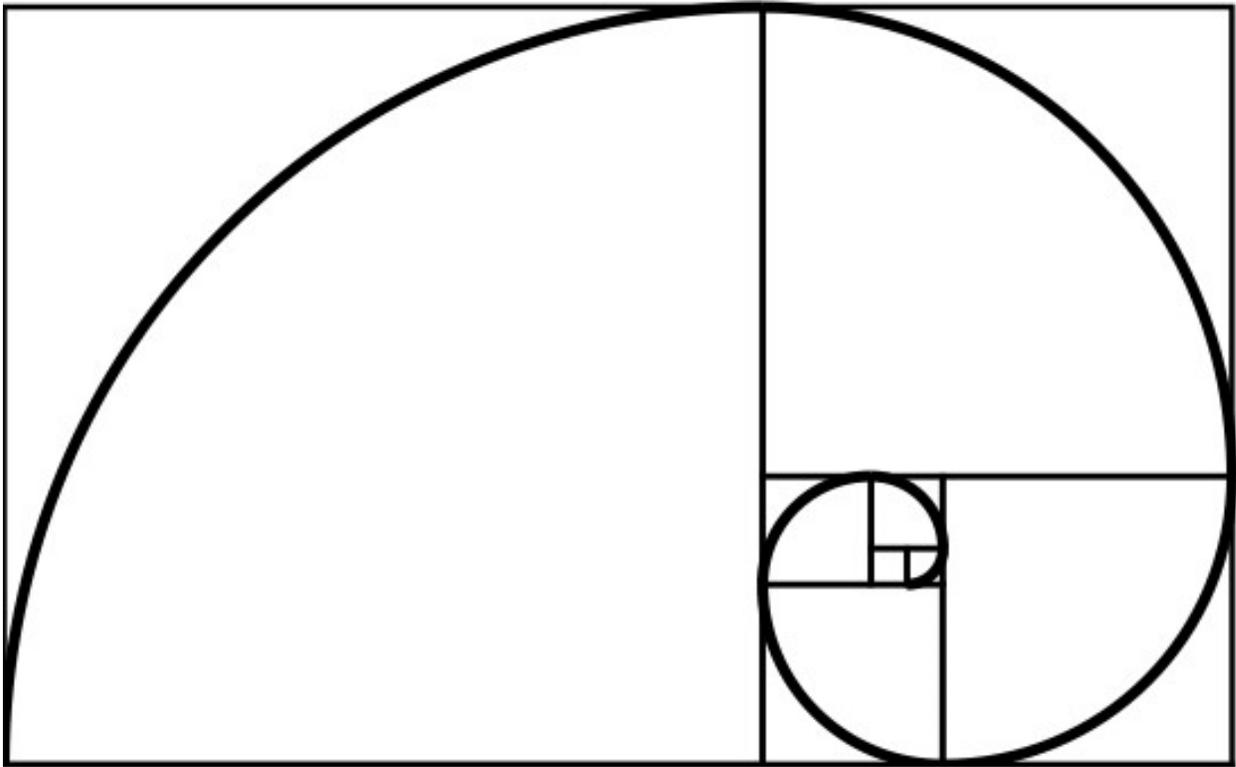
|           | 2 months ago |  | last month |  | current month |  |
|-----------|--------------|--|------------|--|---------------|--|
| Newborns  |              |  |            |  |               |  |
| Teenagers |              |  |            |  |               |  |
| Adults    |              |  |            |  |               |  |

8. We know that any number in the Fibonacci Sequence is equal to the sum of the previous two numbers. Let's figure out how the table with arrows proves this.
- (a) Adult and teenage rabbits are considered to be old rabbits, while newborn rabbits are new rabbits. Looking at the purple lines you have drawn, how are the number of pairs of old rabbits in the current month and the total number of pairs of rabbits from last month related?
  
  - (b) Looking at the green lines you have drawn, how are the number of pairs of new rabbits in the current month and the total number of pairs of rabbits from 2 months ago related?
  
  - (c) Adding up the number of pairs of old rabbits and new rabbits in the current month gives the total number of pairs. Using your answers from part a and b, show that the number of total pairs in the current month equals the sum of the pairs from the previous 2 months.

9. Fibonacci's sequence is also seen in other instances of nature! For example, the spiral of a Nautical Seashell is related to Fibonacci's Sequence!



Assume that the side lengths of the smallest, inside squares are 1 cm. Find the side lengths of the rest of the squares by starting from the center and working outward (following the spiral of the shell).



What do you notice about the side lengths?

10. Create a pattern similar to Fibonacci's Sequence, but instead of starting with 1 pair of newborn rabbits in Month 0, start with 2 pairs. Write down the first 8 numbers in the sequence.
  
  
  
  
  
  
  
  
  
  
11. Assume the first two numbers in Fibonacci's sequence are 1 and 3. Using the relation we found in problem 6, write down the first 8 numbers in the new sequence.