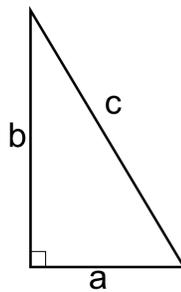


Math Circle
Beginners Group
January 17, 2016
Geometry II

Warm-up Problem

1. How many equilateral triangles can you make using six identical line segments?

Let us now look at the Pythagorean Theorem.



The Pythagorean Theorem states that, in a right triangle with two side lengths a and b and hypotenuse of length c , the following will be true.

$$a^2 + b^2 = c^2$$

Any three numbers that satisfy the above equality form a Pythagorean Triple.

2. Complete the following Pythagorean Triples:

(a) $a = 3, b = 4, c = ?$

(b) $a = 5, b = 12, c = ?$

(c) $a = 6, b = ?, c = 10$

(d) $a = ?, b = 24, c = 25$

3. Can you think of any real-world applications of the Pythagorean Theorem?

Similarity in Triangles and Shadow Geometry

We are now going to learn how to measure the heights of tall objects outside. Imagine that you are an engineer in the 19th century, and you do not have the physical resources to measure the height of a very tall granite wall, except your incredulous wits. This is what you come up with¹:

- You use a straight stick, 10 feet long, as a reference with which the height of the wall will be measured.
- You move approximately 500 feet away from the granite wall, and then affix the stick perfectly vertically in the ground and firmly cement it.
- Then you walk slightly away from the stick at such a distance that lying on the ground, it is possible to see the top of the stick and the top of the wall in one straight line. You carefully mark this point in the ground using a milestone.



- What you have here are two **similar triangles**. Similar figures are figures that have the same shape but not necessarily the same size.

1. Can you spot the two similar triangles in the picture above?

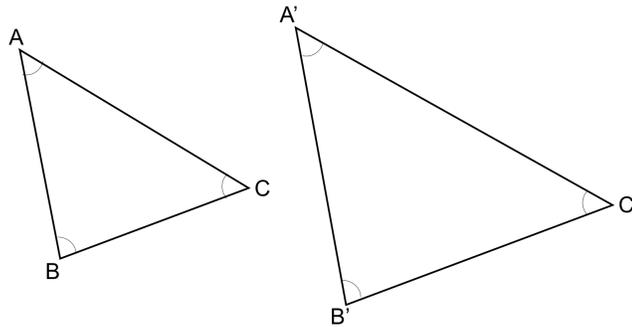
¹The following explanation is an adaptation of an excerpt from “The Mysterious Island” written by Jules Verne in 1874.

- Two triangles can only be similar (have the same shape) if they have angles that are pairwise equal. This means that if $\triangle ABC \sim \triangle A'B'C'$ (“ \sim ” is the sign of similarity),

$$\angle A = \angle A'$$

$$\angle B = \angle B'$$

$$\angle C = \angle C'$$



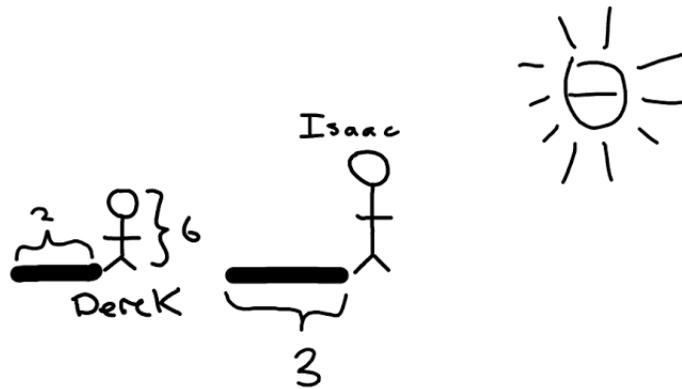
- In addition, two triangles are similar if and only if they have side lengths with pairwise equal ratios. This means that

$$\frac{AB}{A'B'} = \frac{BC}{B'C'} = \frac{CA}{C'A'}$$

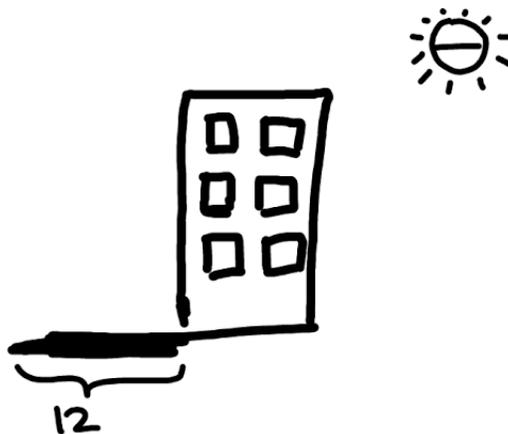
2. Now think back to the granite wall and the 10-foot-long stick. Draw a geometric picture of the two triangles that are similar, and label their sides. Explain why the triangles are similar.

3. Write a proportion in terms of the side lengths of the triangles that you can use to find the height of the wall.

4. If the distance between the milestone and the stick is 15 feet and the distance between the milestone and the wall is 500 feet, what is the height of the wall?
5. Isaac and Derek are measuring the heights of various objects outside. They do not know what time it is, but they notice that Derek is 6 feet tall, and he has a shadow that is 2 feet long.
- (a) Isaac looks at his shadow and notices that it is 3 feet long. Show the two triangles that are similar. How tall is Isaac?

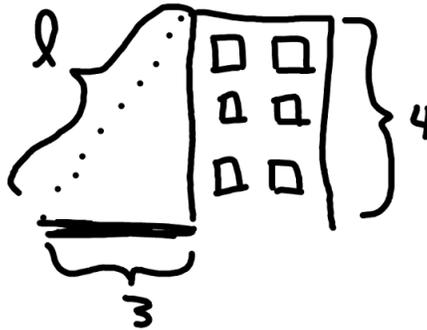


- (b) Isaac and Derek look at a building nearby and notice that its shadow is 12 feet long. How tall is the building?

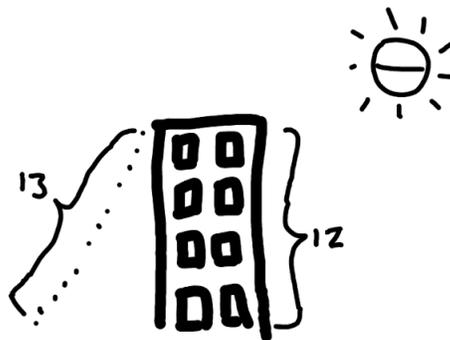


- (c) Isaac dislikes the sun, so he wishes to stand in the shadow of the building. How far away from the building can Isaac stand and still be in the shade?

- (d) Derek loves to be in the sun. As a result, he uses a ladder to get from the bottom of the building to its top. His requirement for placing the ladder is that the entirety of the ladder should be in the sun. Suppose that he wants to climb a building of height 4 units that has a shadow of length 3 units. How long does his ladder need to be?



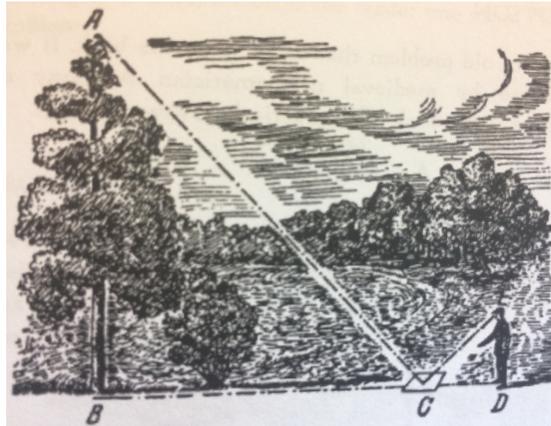
- (e) Derek now wants to scale a building of length 12 units, and he needs a ladder of length 13 units to do so. How long is the shadow of the building?



6. A vertical pole of length $6m$ casts a shadow $4m$ long on the ground and at the same time a tower casts a shadow $28m$ long. Draw a figure and find the following:

- (a) the similar triangles. Explain why the triangles are similar.
- (b) the height of the tower

7. Here is a different way of determining the height of a tree using a mirror. At some distance from the tree to be scaled, place a mirror horizontally on the ground at point C . Move away from the mirror to a point D , while standing at which the top of the tree (point A) lies at your eye level. (Hint: Light reflects at the same angle as it reaches the mirror. This is called the Law of Reflection.)

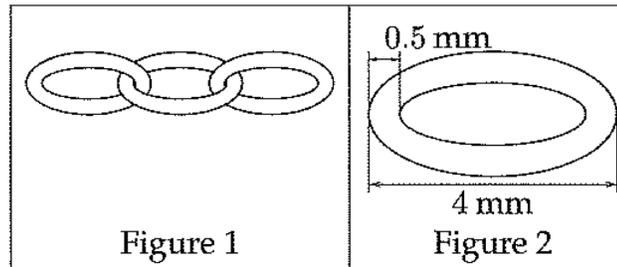


(a) Show which triangles are similar and explain why.

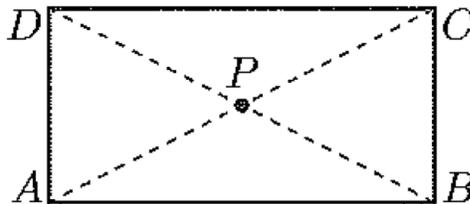
(b) If your height is 6 feet, the mirror is 4 feet away from you, and you are 32 feet away from the tree, what is the height of the tree?

Math Kangaroo Problems²

1. A jeweler can make chains of any length using identical links. Figure 1 shows such a chain made out of three links. A single link is shown in figure 2. What is the length of a chain made out of seven links?

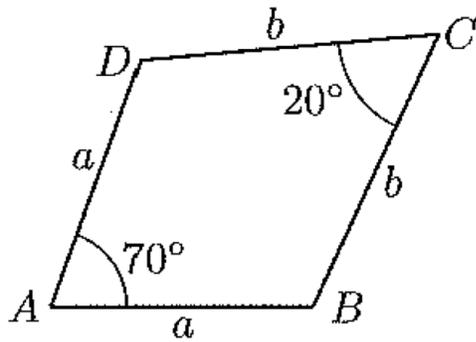


2. Rectangle $ABCD$ has a perimeter of 120cm and its diagonals intersect at point P . The distance from P to side BC is twice the distance from P to side AB . What is the area of the rectangle?



²The problems are taken from the 2010 Math Kangaroo contest.

3. What is the measure of $\angle ABC$ in the quadrilateral $ABCD$ shown in the picture?



4. What fraction of the given square is the shaded region?

