Perform Math in your Head

Mental Math Tips and Techniques

Part 1: Basic Mental Math

Adding, subtracting, multiplication, and division can all be done very quickly in your head if you know the tricks to it. Here are some helpful tips for doing basic mental math:

1. When adding or subtracting, it's easiest to move from left to right instead of right to left. When values go over 10, simply add 1 to the previous digit.
2. When adding and subtracting numbers close to an easy number to add and subtract (example: 593 is close to 600), add or subtract the difference from the value and the easy number and perform the opposite operation on the other number for addition, and the same operation for subtraction.
   - For example, is we were asked to find 593 + 680, we would add 7 to 593 to get 600 and subtract 7 from 680 to get 673. Now, we can easily see the sum is 1,273.
   - If instead, we were asked to perform 593 – 444, we can add 7 to both the minuend and the subtrahend to get 600 – 451. From here, it's easy to see our difference is 149.
3. When multiplying values, also try to move to an easy multiplication number (ending in a zero), and simply add or subtract the numbers you have left.
   - For example, when asked to multiply 89 and 6, it is much easier to multiply 90 by 6 and subtract off 6.

Part 2: Techniques on Squaring Numbers and Using Squares in Other Forms of Multiplication

1. Squaring numbers that end in 5 is very quick. Assume a number can be written as n5, where n is the numbers preceding 5. $n5^2$ can be found by multiplying n by n+1 and writing 25 to the right of the answer. For example, 75^2 is 5,625, which is 7×8 with a 25 written on the right.

2. Don't neglect the power of changing a number n into $(a+b)^2$. Here is an example of using this technique when asked to find 57^2.

   - Notice that 57 is close to 60. Therefore, we can write 57^2 as $(60 – 3)^2$.

   - From here, we can expand that binomial into $60^2 - 60 \times 3 \times 2 + 3^2$

   - Now, we just do simply square calculations and addition to find the answer: 3,249.

3. Squaring numbers that are large is easy by splitting up our numbers to make them more manageable. Here is an example with 44^2.

   -First, find the closest number that ends in a ten and calculate the difference between the number that will be squared and that number: 44 is closest to 40 with a difference of 4

   -Next, add that difference to the number that must be squared: $44 + 4 = 48$

   -Multiply these two numbers together and add the difference squared—this is your answer: $40 \times 48 + 4^2 = 1{,}936 = 44^2$

   Prove why this is true. In a general form, given some number n that we want to square, and a difference s that turns n into a nicer number, we have a formula $n^2 = (n-s)(n+s) + s^2$ to get an answer.

4. We can use the previous formula to work backwards too. If we are given two numbers that must be multiplied together and are both equidistant from an easy square n, we can use the formula to find our answer. In the general form, this can be written as $(n – s)(n + s) = n^2 – s^2$. For example, 33×27 is $30^2 – 3^2$, or 891.

5. When multiplying, we can use our tricks of addition—if we divide one number by a value, we can multiply the other number by that value and still get the same answer. This is useful for bringing two numbers closer together to use our formulas relating to squares. Here is an example of this using the expression 88×18

   - Multiply 18 by 2 while simultaneously dividing 88 by 2—this gives us the values 44 and 36.

   - Suddenly, this becomes a much easier multiplication problem building off what we know before: 36×44 is $40^2 – 4^2$, or 1,584.

6. Multiplying by 11 is easy. A number multiplied by 11 will always gain one digit (or two, if it is a number that begins with at least the digits of 91) and, if the product is laid underneath the original number, the leftmost digit stays the same, and each digit is the sum of the two digits above it. If a value exceeds 10, add one to the value directly on the left. *Prove why this works.* This is confusing so here's an example: 5,113,601 × 11 =

Original		5	1	1	3	6	0	1
Original × 11	5	6	2	4	9	6	1	1

Note that, when written like this, every digit in the product is the sum of the two digits above it, unless they are on the end, in which case they simply take the value of the one digit above. Here is another example with values that go over 9.

Original		6	7	1	9	6	2
Original × 11	7	3	9	1	5	8	2

And a final example where the original value begins with values greater than 91.

Original			9	1	8	9	0	3
Original × 11	1	0	1	0	7	9	3	3

<u>Part 3: Other Tricks</u>

1. Estimating Square Roots—Divide the given value by the number that is the square root of the closest perfect square. (for example, divide 55 by 7, as 7^2 is 49). Now, take the quotient and average it with the divisor—this is your estimate. Using the previous example:

   - If we are asked to estimate the square root of 55, we note that 49 is the closest perfect square, and divide 55 by it.

   - Now, we average 55/7 and 7 to get 52/7, which is 7.42857.

   - The actual square root of 55 is 7.41620, which is quite close—only slightly more than a hundredth away.

   Prove why this method works—and would it only work with perfect squares?

2. When multiplying, the number of digits in your answer should be the number of digits in the first number plus the number of digits in the second number, possibly minus 1. When dividing, the number of digits in your answer should be the number of digits in the dividend minus the number of digits in your divisor, possibly plus 1. For example: a number with three digits multiplied by one with six will either have eight or nine digits in it.
3. A trick when comparing the combination of multiple fractions is to use the *less than* or *greater than* notation, usually noted as $^{+}$ or $^{-}$after the number. As an example, which of these sums is greater: + or + Using our notation, we can rewrite this as $^{+} + ^{+}$ and $^{-} + ^{-}$. When rewriting it like this, we can see the first sum will be greater, as both of the summands are slightly greater than 0.5, while both the summands of the second sum are slightly less than 0.5.