Circumcenter of Mass

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Exercises

1. Prove that for any nondegenerate triangle, there is a unique point equidistant from its vertices.

2. In the standard basis, what is the expression for the matrix J which rotates a vector in $\mathbb{R}^2$ by 90 degrees clockwise?

3. Let C be a center of a triangle Δ. Show that if Δ is isosceles, C must lie on the line of symmetry of Δ.

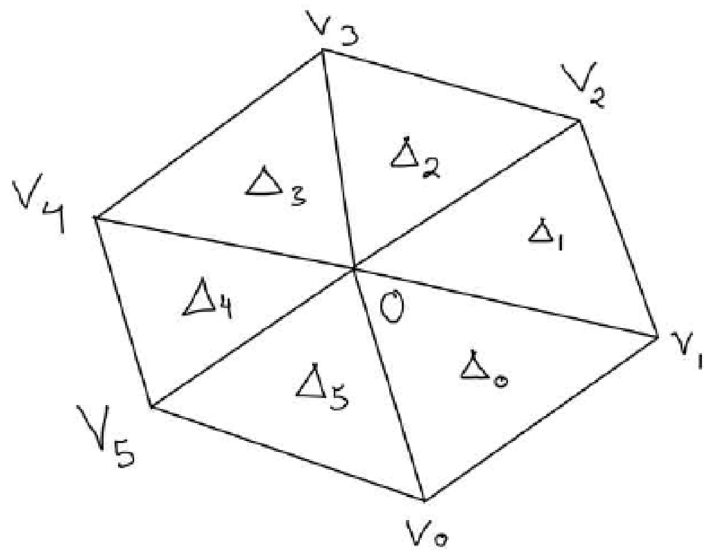
4. Let $V = (V_0, V_1, \ldots, V_{n-1})$ be a polygon in the plane. Consider the expression

$$\mathrm{CCM}(V) := \frac{1}{4A(V)} \sum_{i=0}^{n-1} |V_i|^2 J(V_{i+1} - V_{i-1}).$$

Here $A(V)$ is the oriented area of V. Show that this expression commutes with rescalings, reflections and translations of V.

5. For a triangle Δ, interpret $C(\Delta)$ geometrically. (Hint: see title).

6. Triangulate a polygon $V = (V_0, V_1, \ldots, V_n)$ radially through a point O as in the figure. What is the relation between the $C(\Delta_i)$ and $C(V)$?



7. Let V be a polygon with sides satisfying $|V_0V_1| = |V_1V_2| = \ldots = |V_{n-2}V_{n-1}|$ (but not necessarily equal to $|V_{n-1}V_0|$). Use the fact that CCM and CM coincide for an equilateral polygon together with the Archimedes' lemmas to show that the Euler line is orthogonal to side $V_{n-1}V_0$.