Probability and the Monty Hall Problem

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Warm-up:

There is a sequence of number:

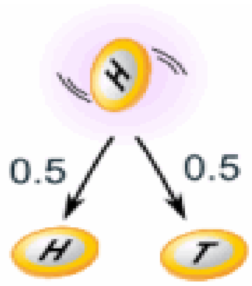
1, 2, 4, 8, 16, 32, 64, …

How does this sequence work? How do you get the next number from the previous one?

Notation of Probability:

The probability of A can be denoted as P(A), where

$$P(A)=\frac{number\ of\ outcomes\ being\ A}{total\ number\ of\ outcomes}$$



1. Suppose you flip a coin, and we denote getting heads as H and getting tails as T.
   a) What is the probability that you get head if you flip a coin once?

   b) List all outcomes when a coin is flipped twice. (For example, TH stands for tail, head)

c) What is the probability of getting H on the second flip (disregard the result of the first flip)?

d) List all outcomes when a coin is flipped three times.

e) What is the probability that you get TTH?

f) Without listing all possible outcomes, can you find the probability of getting HTHTT after 5 flips?

2. Suppose you have a spinner numbered 1 to 7, and a coin.
   a) What is the probability of getting an odd number on the spinner?

b) List all outcomes when you spin the spinner and toss the coin at the same time. (For example, 1T stands for getting an 1 on the spinner and a tail.)

c) Circle the outcomes where the number is odd and the coin shows Tail.

d) What is the probability that you get an odd number on the spinner and a tail on the coin?

3. A jar contains 2 red balls, 3 green balls, 4 white balls and 5 yellow balls.

a) One ball is randomly chosen. How many different outcomes are there?

b) What is the probability that we choose a green ball?

c) After we put the green ball back, what is the probability that we choose a green ball again when we randomly choose a ball for the second time?

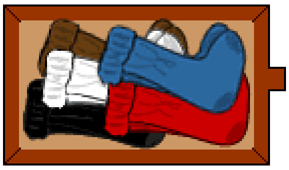
d) When two balls are chosen randomly from the jar, and the first ball is put back into the jar before the second ball is chosen. What is the probability that both balls chosen are green?

4. If the probability that an apple will be ripe 2 weeks is $\frac{1}{2}$ and the probability that a banana will be ripe in 2 weeks is $\frac{1}{3}$, what is the probability that both the apple and the banana will be ripe in 2 weeks?

Independent event probability:

Two events, A and B, are independent if the fact that A occurs (or does not occur) does not affect the probability of B occurring. At the same time, the fact that A occurs (or does not occur) does not affect the probability of B occurring. When the probability that A occurs is P(A), the probability that B occurs is P(B), for independent events, the probability that both of them occur is

$$\mathbf{P(A\ and\ B)= P(A) \times P(B)}$$



1. Let's say you have a drawer of marbles, 3 blue marbles, 3 red marbles, 2 black marbles, 2 brown marbles and 1 white marble.
   a) What's the probability that you pick a blue marble without looking?

   b) Assume that you pick a blue marble on the first try and do not put it back, what's the probability that you pick another blue marble without looking on the second try?

   c) Assume that you pick a blue marble on the first try and another blue marble for the second try, if both times you do not put the marbles back to the drawer, what's the probability that you pick a blue marble again on the third try?

d) From what we get in a), b) and c), what is the probability that you pick three marbles from the drawer and they are all blue?

2. In a shipment of 10 computers, 3 are defective (of low quality). Three computers are randomly selected and tested.

a) What is the probability that the first computer chosen is defective?

b) Assume the first computer chosen is defective, what is the probability that the second computer chosen is also defective?

c) Assume both the first and second computers chosen are defective, what is the probability that the third computer chosen is defective?

d) What is the probability that all three computers are defective if the first and second ones are not put back after being tested?

3. A purse contains four $5 bills, five $10 bills and three $20 bills. Two bills are selected. After the first bill is selected, it is not put back into the purse.

a) List all the outcomes we can have for the two bills selected. (For example, if $5 and $10 are selected, it can be denoted as 5,10)

b) Circle the outcomes where both bills selected are $5.

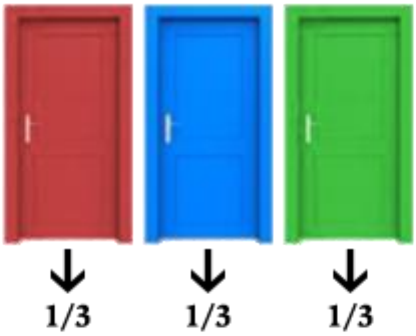
c) What is the probability that both bills selected are $5?

Dependent event probability:

Two events, A and B, are dependent if the outcome or occurrence of one of them affects the outcome or occurrence of the other.

The Monty Hall Problem- Let’s play a game!

1. The Monty Hall Problem gets its name from the TV game show, *Let's Make A Deal*, hosted by Monty Hall. Imagine the following: there are three doors in front of you, and you are given the opportunity to select one closed door of three. There is a **prize** hiding behind one of the three doors. The other two doors hide goats. (Remember: the prize is much more valuable than the goats!) Once you have made your selection, Monty Hall will open one of the remaining doors, revealing that it does not contain the prize. You then have the choice to switch your choice of door, or to stay with your original choice. In order to maximize the chances of winning the prize, do you switch or not? Discuss with your classmates and make the decision.



2. Let’s think about another problem first. Imagine there are 100 doors here in front of you. There is, again, one **prize** behind one of the 100 doors. The rest of the doors have goats behind. And you have the opportunity to select one closed door. After you make your selection, say, Door No. 1, I will kindly tell you that Door No.2 is not the door with the prize behind. I

will also tell you that Door No.3 is not the door with the prize behind. Neither is Door No.4, Door No. 5, Door No. 6… Door No. 98, Door No.99. So now out of my kindness, I have helped you eliminate 98 wrong choices, but you still don't know which door, Door No.1 or Door No.100, is the door with the prize.

a) What is the probability of you picking the prize door at first?

b) What is the probability of the prize staying behind the other 99 doors? (Hint: think of them as a group)

c) After elimination of 98 wrong doors, what is the probability of the door you originally picked being the right door? (Does it change?)

d) What is the probability of the prize staying behind the other door? (Do you still remember how we consider the 99 doors as a group?)

e) Stay or switch? Which door has bigger probability of winning?

f) Do you switch or stay with the original pick when there are 50 doors instead? After elimination of 49 doors, what is the probability of your original pick being the prize door? How about the probability of the other door left being the right one? (Think about the 49 doors as a group!)

Now let's go back to our original Monty Hall problem and let's try to win the prize!

a) What is the probability of you picking the prize door at first?

b) What is the probability of the prize staying behind the other two doors? (Hint: think of them as a group)

c) After elimination of one door (i.e., Monty Hall opens one of the remaining doors, showing there is no prize behind), what is the probability of the door you originally picked being the right door? (Does it change?)

d) Then what is the probability of the prize staying behind the other door? (Think about the problem before with 100 doors)

e) Stay or switch? Which door has bigger probability of winning?