LAMC Intermediate Circle **January 10, 2016**

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Warm-up

Problem 1 *Find a fraction p/q such that*

- *the integers p and q are co-prime;*
- *the fraction is proper, $0 < p/q < 1$; and*
- *the fraction does not change when one adds 30 to the numerator and 40 to the denominator.*

Problem 2 *Jack and Jill have an equal amount of money. How much money should Jack give Jill so that the girl has five times as much as the boy?*

Problem 3 *Prove that* $\log_b xy = \log_b x + \log_b y$.

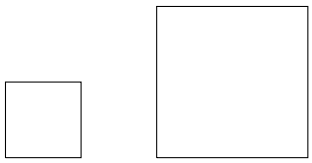
Problem 4 *Prove that* $\log_b x^y = y \log_b x$.

Problem 5 *Find all the (real) solutions of the following equation.* $\log_9^2 x^2 = 1$

What is dimension?

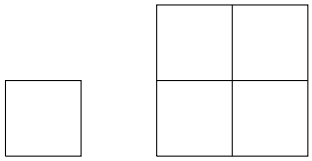
In what follows, we give one possible answer to this question.

Let us take a square and increase its linear size twice.

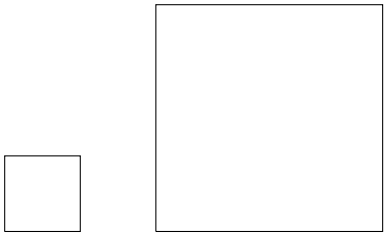


Question 1 *How many original squares does one need to tile the enlarged square?*

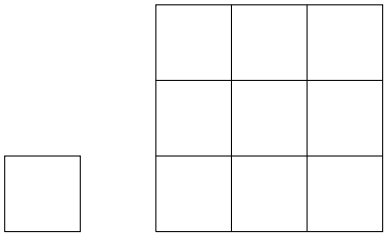
The answer is so very obvious – just look at the picture and count ... four!



Let's try again. Let us take a square and increase its linear size three times.



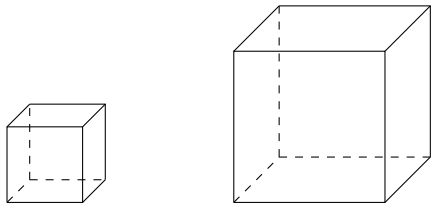
Question 2 *How many original squares does one need to tile the enlarged square?*



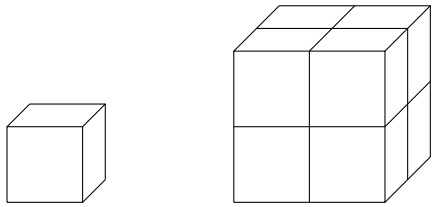
Problem 6 *One takes a square and enlarges it k times where k is a positive integer. How many original square does one need to tile the enlarged square?*

Question 3 *Do you see the dimension of the square appearing in your solution to Problem 6? What is the dimension of the square?*

This time, let us take a cube and increase its linear size twice.



Question 4 *How many original cubes does one need to tile the enlarged cube?*



Problem 7 *One takes a cube and enlarges it k times where k is a positive integer. How many original cubes are needed to tile the enlarged cube? According to this approach to dimension, what is the dimension of the cube?*

Problem 8 *Draw a hypercube (4D cube) in the space below.*

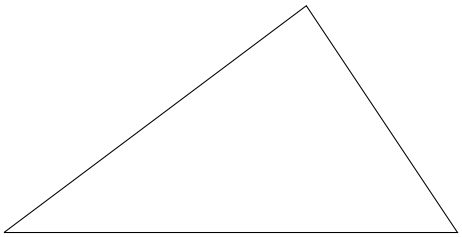
Problem 9 *One takes a hypercube and enlarges it three times. How many original hypercubes does one need to tile the enlarged hypercube?*

Problem 10 *One takes a hypercube and enlarges it k times where k is a positive integer. How many original hypercubes are needed to tile the enlarged hypercube? What is the dimension of a hypercube?*

Problem 11 *One takes an n-dimensional (n-dim) cube and enlarges it 4 times. It takes 16,384 original n-dim cubes to tile the enlarged n-dim cube. What is the dimension n?*

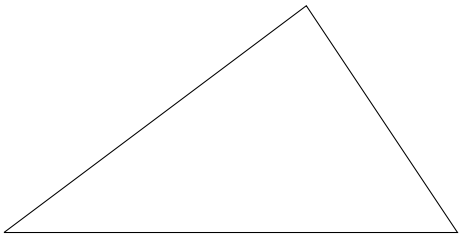
Problem 12 *One takes an n-dimensional cube and enlarges it k times where n and k are positive integers. How many original n-dim cubes does one need to tile the enlarged n-dim cube?*

Problem 13 *Use a compass and a ruler (straightedge) to construct a triangle having the linear size twice more than the triangle below.*



Problem 14 *Use a compass and a ruler to construct an accurate tiling of the larger triangle from Problem 13 with the smaller ones. How many triangles are there in the tiling?*

Problem 15 *Use a compass and a ruler (straightedge) to construct a triangle having the linear size three times as much as the triangle below.*



Problem 16 *Use a compass and a ruler to construct an accurate tiling of the larger triangle from Problem 15 on the previous page with the smaller ones. How many triangles are there in the tiling?*

Problem 17 *How many original triangles does one need to tile the triangle having the linear size k times larger than the original triangle?*

Problem 18 *Based on the five problems above, what is the dimension of a triangle?*

Problem 19 *Draw a pentachoron (4D tetrahedron) in the space below.*

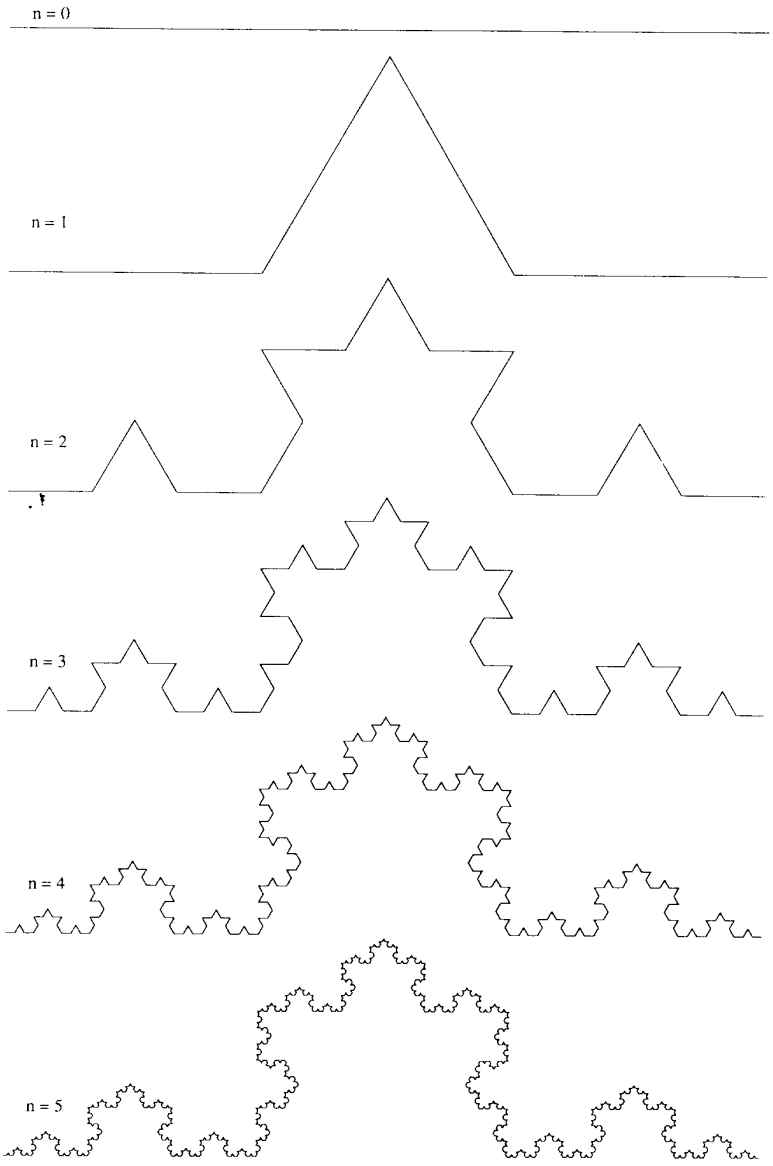
Problem 20 *How many original pentachorons does one need to tile the pentachoron having the linear size* $2k$ *times larger than the original 4D solid?*

Question 5 *What is heavier, one pound of lead or one pound of goose down? Why?*

Question 6 *What is the average density of an object?*

Problem 21 *The average density of a die-cast 1:200-scale model airplane is ten times greater than the average density of the original. How many times is the original plane heavier than the model?*

Recall that the limit of the following procedure is known as the *Koch curve*. The finite steps of the construction are known as the Koch curves of order $n = 0, 1, 2, \ldots$



Problem 22 *The base length of the Koch curve is increased three times. How many Koch curves of the original base length are needed to tile the new one?*

Problem 23 *Find the dimension of the Koch curve.*

If you are finished doing all the above, but there still remains some time...

Problem 24 *Four different rays are drawn from one point in the plane. The rays split the plane into four angles numbered one through four in the clock-wise direction. The first angle is congruent to the third one. Prove that the angle bisectors of the second and fourth angle together form a straight line.*