

# The Gini Index

LA Math Circle

High School II

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Based on notes by Robert Brown

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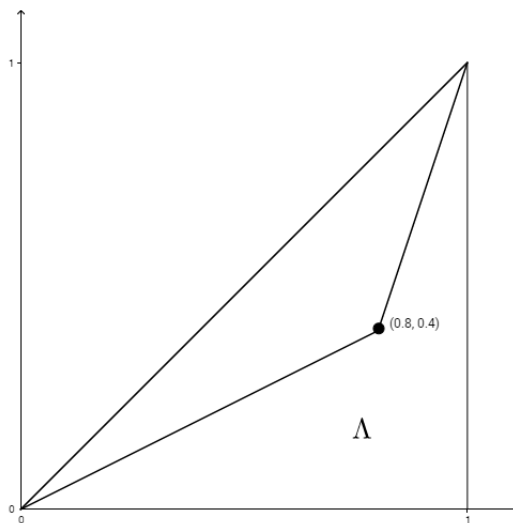
## **Lorenz curve and Gini index**

1. Why do the points  $(0, 0)$  and  $(1, 1)$  lie on the Lorenz curve?
2. Explain why the line segment from  $(0, 0)$  to  $(1, 1)$  should be called the “equal distribution line”.
3. Why does the Lorenz curve lie below the equal distribution line? For instance, why can't  $(.25, .75)$  lie on the Lorenz curve?

4. What distribution of income would make the area of  $\Gamma$  equal  $\frac{1}{2}$ ?

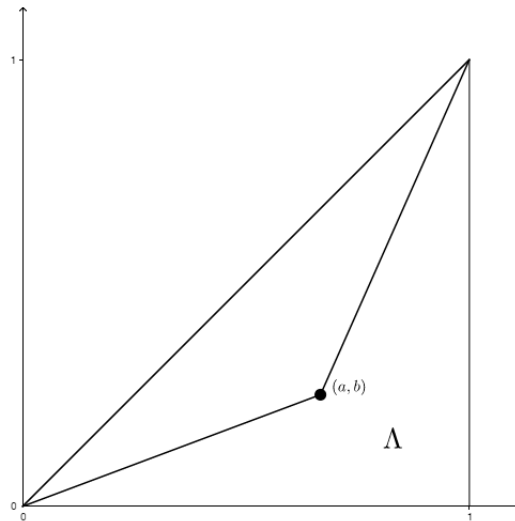
5. Given the Lorenz curve consisting of two line segments in the figure, calculate the area of  $\Lambda$  and use that information to compute the Gini index.

Figure for Problem 5



6. Calculate the area of  $\Lambda$  in the figure and then write a general formula for this “one-point estimate” of the Gini index in terms of  $a$  and  $b$ .

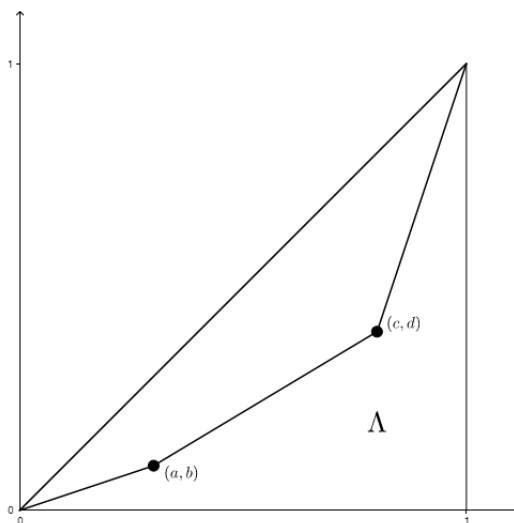
Figure for Problem 6



7. If we base our calculation of the Gini index on a one-point estimate, will that estimate be larger or smaller than the actual value of the Gini index? Why?

8. Calculate the area of  $\Lambda$  in the figure and then write a general formula for this “two-point estimate” of the Gini index in terms of  $a$ ,  $b$ ,  $c$ , and  $d$ .

Figure for Problem 8



9. Use the answer to Problem 8 to make a two-point estimate of the Gini index if  $(a, b) = (.8, .4)$  and  $(c, d) = (.99, .8)$ .

10. Consider a country with  $n$  people. Let the incomes of all the people in the country be  $y_1, \dots, y_n$ , sorted in ascending order, and let  $Y = y_1 + \dots + y_n$ .

a. Why is the Gini index

$$G = \frac{2}{n} \sum_{k=1}^n \left( \frac{k}{n} - \frac{y_1 + \dots + y_k}{Y} \right)?$$

b. Show that

$$G = \frac{1}{n} \sum_k (2k - (n+1)) \frac{y_k}{Y}.$$

c. Show that

$$G = \frac{\sum_{i,j=1}^n |y_i - y_j|}{2nY},$$

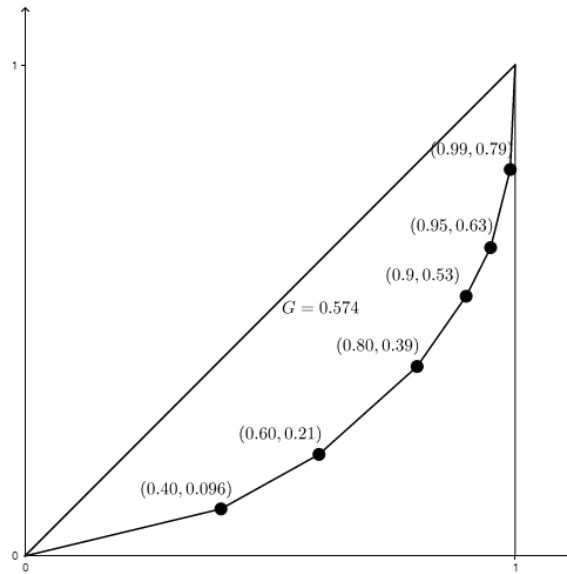
even if  $y_1, \dots, y_n$  are not necessarily sorted in ascending order.

d. The city of Villestown has four people, Alice, Bob, Carl, and Danielle. Their annual incomes are \$20000, \$40000, \$60000, are \$80000, respectively. Find the Gini index for Villestown.

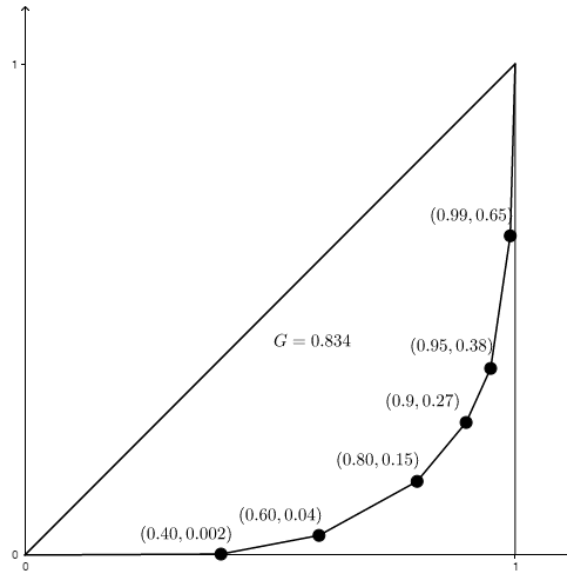
## Some limitations of the Gini index

11. Give examples to demonstrate that two income distributions which are qualitatively very different can have the same Gini index.
  
  
  
  
  
  
  
  
  
  
  
  
  
  
  
  
12. (*Households vs. individuals.*) In Villestown (see previous page), Alice and Bob form a household and Carl and Danielle form a household. Considering the *household* income distribution rather than the *individual* income distribution, what is the Gini index?
  
  
  
  
  
  
  
  
  
  
  
  
  
  
  
  
13. Give examples to show that even if everyone gets richer, the Gini index may increase, and even if everyone gets poorer, the Gini index may decrease.

Lorenz curve for income distribution in the US, 2006<sup>1</sup>



Lorenz curve for wealth distribution in the US, 2007<sup>2</sup>



<sup>1</sup>Wolff, Edward N. (2010). Recent Trends in Household Wealth in the United States: Rising Debt and the Middle-Class Squeeze—an Update to 2007. Levy Economics Institute. [http://www.levyinstitute.org/pubs/wp\\_589.pdf](http://www.levyinstitute.org/pubs/wp_589.pdf)

<sup>2</sup>ibid.