

# TRANSFORMATIONS VIA PERMUTATIONS

Junior Circle

11/22/15

We will look at the following transformations of an equilateral triangle:

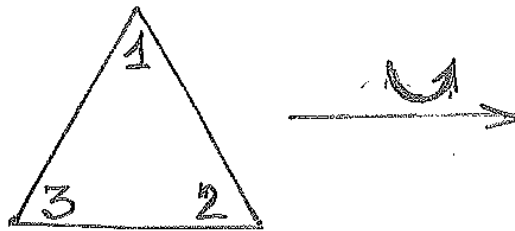
- Rotations;
- Reflections (flips) across a line;

The two types of rotation are:

- Clockwise rotation  $\circlearrowright$ :



- Counterclockwise rotation  $\circlearrowleft$ :



(1) When the triangle is rotated, the vertices end up in the new places. This way, we get a permutation of vertices:

- The first row is starting positions;
- The second row is ending positions;

Write down the permutations corresponding to the clockwise and the counterclockwise rotations:

(a) Clockwise rotation  $\circlearrowright$ :

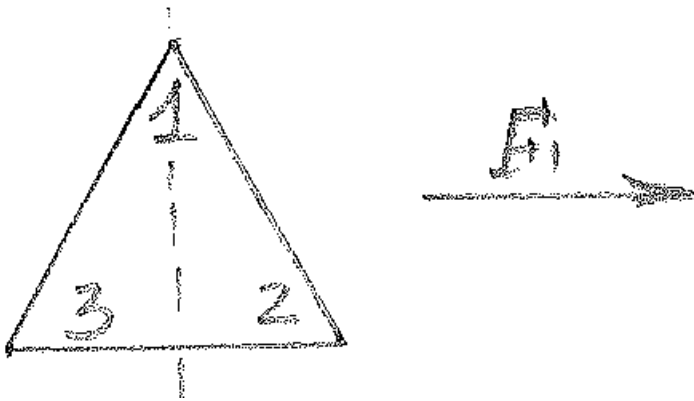
$$\begin{pmatrix} 1 & 2 & 3 \\ \downarrow & \downarrow & \downarrow \end{pmatrix};$$

(b) Counterclockwise rotation  $\circlearrowleft$ :

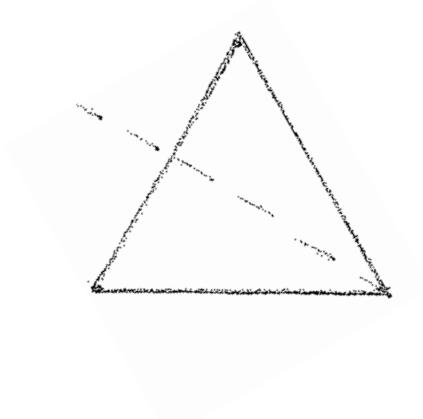
$$\begin{pmatrix} 1 & 2 & 3 \\ \downarrow & \downarrow & \downarrow \end{pmatrix};$$

There are also three flips:

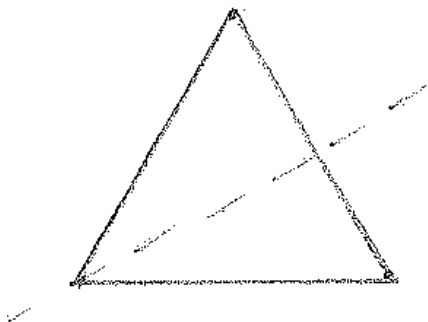
- The flip  $F_1$  through line going through 1 and switching 2 and 3:



- The flip  $F_2$  through line going through 2 and switching 1 and 3:



- The flip  $F_3$  through line going through 3 and switching 1 and 2:



(2) When the triangle is flipped, the vertices also end up in the new places. Write down the permutations corresponding to the clockwise and the counterclockwise rotations:

(a) Flip  $F_1$ :

$$\begin{pmatrix} 1 & 2 & 3 \\ \downarrow & \downarrow & \downarrow \end{pmatrix};$$

(b) Flip  $F_2$ :

$$\begin{pmatrix} 1 & 2 & 3 \\ \downarrow & \downarrow & \downarrow \end{pmatrix};$$

(c) Flip  $F_3$ :

$$\begin{pmatrix} 1 & 2 & 3 \\ \downarrow & \downarrow & \downarrow \end{pmatrix};$$

When no transformation is performed, we get the *identity permutation*:

$$\begin{pmatrix} 1 & 2 & 3 \\ \downarrow & \downarrow & \downarrow \\ 1 & 2 & 3 \end{pmatrix};$$

(3) Let's find the result of performing two transformations in a row:

(a) Find what transformation  $\circ \circ$  equals to in two different ways:

- Label vertices and write down what the resulting transformation is:



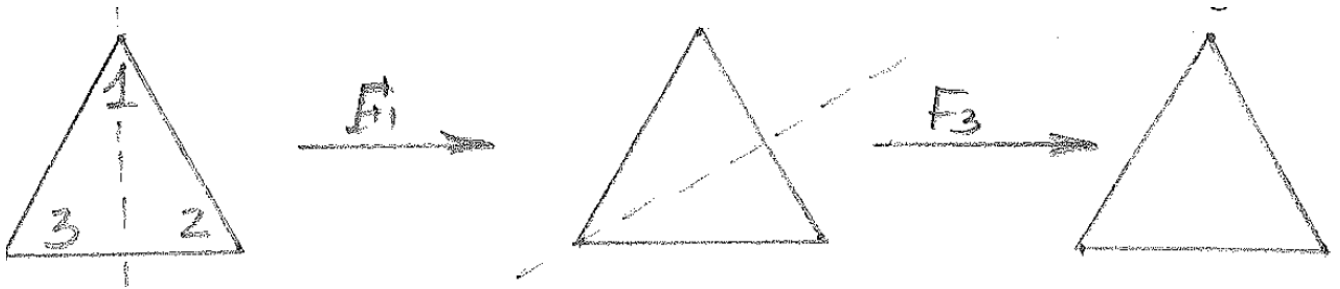
- Multiply permutations:

$$\begin{pmatrix} 1 & 2 & 3 \\ \downarrow & \downarrow & \downarrow \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 & 3 \\ \downarrow & \downarrow & \downarrow \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 \\ \downarrow & \downarrow & \downarrow \end{pmatrix};$$

- Are the results you get when using the picture and when multiplying permutations the same?

(b) Find what transformation  $F_1 \circ F_3$  equals to in two different ways:

- Label vertices and write down what the resulting transformation is:

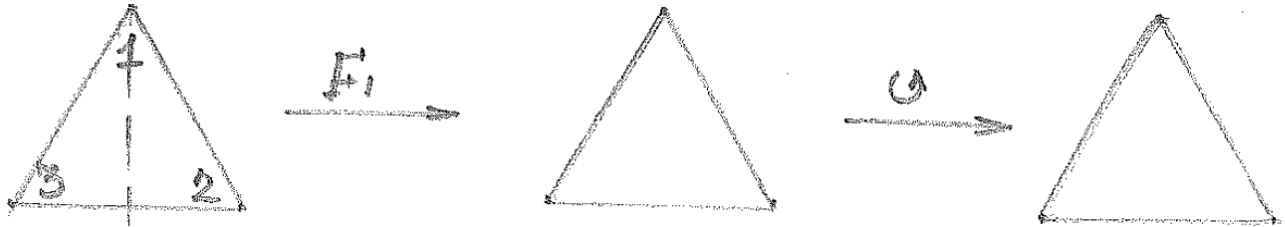


- Multiply permutations:

$$\begin{pmatrix} 1 & 2 & 3 \\ \downarrow & \downarrow & \downarrow \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 & 3 \\ \downarrow & \downarrow & \downarrow \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 \\ \downarrow & \downarrow & \downarrow \end{pmatrix};$$

- Are the results you get when using the picture and when multiplying permutations the same?
  
  
  
  
  
  
  
  
  
  
- Are the results the same if you switch the order in which you perform the operations?

- (c) Find what transformation  $F_1 \circ \sigma$  equals to in two different ways:
- Label vertices and write down what the resulting transformation is:



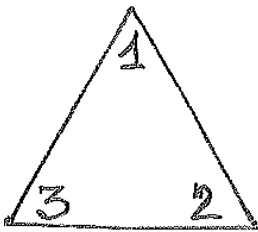
- Multiply permutations:





$$\begin{pmatrix} 1 & 2 & 3 \\ \downarrow & \downarrow & \downarrow \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 & 3 \\ \downarrow & \downarrow & \downarrow \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 \\ \downarrow & \downarrow & \downarrow \end{pmatrix};$$

- Are the results you get when using the picture and when multiplying permutations the same?
  
- Are the results the same if you switch the order in which you perform the operations?

- (4) Fill out the following multiplication table (you can either draw pictures; or flip and rotate a model triangle, or multiply permutations).

Start with this triangle and draw the final triangle in the box. Use the scratch paper at the back if you need it.



	$I$	$F_1$	$F_2$	$F_3$		
$I$						
$F_1$						
$F_2$						
$F_3$						
						
						

(5) Write down as many interesting things about this multiplication table as you can.

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