

Some Statements to Prove

Prove each of the statements below. Be sure to explain all the logical steps of your reasoning. You may (and will need to) assume some things we've learned previously. The goal is to clearly state everything you use.

1. If $a \mid b$ and $b \mid c$, then $a \mid c$.
2. If $a \mid m$ and $a \mid n$, then $a \mid m + n$.
3. If d is a positive integer and $d \mid 1$, then $d = 1$. (Hint: Use prime factorizations.)
4. Prove that for every positive integer n , $n^2 - 1$ is not divisible by 3.
5. Prove that if n is odd, then $n^2 + 1$ is divisible by 8.
6. Prove that if $m \equiv a \pmod{k}$ and $n \equiv b \pmod{k}$ then $mn \equiv ab \pmod{k}$.
7. Prove that if $am + bn = k$, then $\gcd(m, n) \mid k$.
8. Prove that if $am + bn = 1$, then $\gcd(m, n) = 1$.
9. Prove that $d = \gcd(m, n)$ if and only if $d \mid m$ and $d \mid n$, and for every other number d' such that $d' \mid m$ and $d' \mid n$, we also have $d' \mid d$.