QUARTER REVIEW

BEGINNERS NOVEMBER 22, 2015

Warm Up Problem

Simplify the following expressions. Then evaluate the simplified expression using the given values.

(1) $5x^2 + 9x - 4x^2 - 8$

$= 5x^2 - 4x^2 + 9x - 8$

$= x^2 + 9x - 8$

- Evaluate the expression above for $x = 5$.

$x^2 + 9x - 8 = (5)^2 + 9(5) - 8$

$= 25 + 45 - 8 = 62$

(2) $\frac{x^{1234} - x^{1231}}{x^{1230}}$

$= \frac{\cancel{x^{1230}}(x^4 - x^1)}{\cancel{x^{1230}}} = x^4 - x$

- Evaluate the expression above for $x = 10$.

$x^4 - x = 10^4 - 10 = 9990$

(3) $\frac{a^3b^{15}}{b^{16}}$

$$= \frac{a\times a\times a\times \overbrace{\cancel{b\times\cdots\times b}}^{15\ b's}}{b\times \underbrace{\cancel{b\times\cdots\times b}}_{15\ b's}} = \frac{a\times a\times a}{b} = \frac{a^3}{b}$$

- Evaluate the expression above for $a = 2$ and $b = 8$.

$$\frac{a^3}{b} = \frac{2^3}{8} = \frac{8}{8} = 1$$

(4) $2x \cdot \frac{\frac{1}{x}+y}{4}$

$$= \frac{2x(\frac{1}{x}+y)}{4} = \frac{\frac{2x}{x}+2xy}{4} = \frac{2+2xy}{4} = \frac{1+xy}{2}$$

- Evaluate the expression above for $x = 2$ and $y = 3$.

$$\frac{1+xy}{2} = \frac{1+2\times 3}{2} = \frac{7}{2}$$

(5) $ab \cdot (\frac{1}{a} + \frac{2}{9}b)$

$$= \frac{ab\cdot 1}{a} + ab\,\frac{2}{9}b$$

$$= \frac{ab}{a} + \frac{2}{9}ab^2$$

$$= b + \frac{2}{9}ab^2$$

- Evaluate the expression above for $a = 5$ and $b = 3$.

$$b + \frac{2}{9}ab^2 = 3 + \frac{2}{9}\times 5\times 3^2 = 3 + 10 = 13$$

Express y in terms of x. That is, $y = \ldots$, where the expression on the right involves only x. Assume that $x \neq 0$.

(1) $x + y = x^2$

$y = x^2 - x$

(2) $2xy = x^2$

$y = \frac{x^2}{2x} = \frac{x \cdot \cancel{x}}{2\cancel{x}} = \frac{x}{2}$

$y = \frac{x}{2}$

(3) $xy = x + x^2$

$y = \frac{x + x^2}{x} = \frac{x}{x} + \frac{x^2}{x} = 1 + x$

$y = x + 1$

(4) $(x + 3y)(2x + y) = 3y^2$

$x(2x+y) + 3y(2x+y) = 3y^2$

$2x^2 + xy + 6xy + \cancel{3y^2} = \cancel{3y^2}$

$2x^2 + 7xy = 0$

$7xy = -2x^2$

$y = \frac{-2x^2}{7x} = \frac{-2}{7}x$

$y = \frac{-2}{7}x$

(5) Factor $91x^3 - 13x^5$.

$= 13x^3(7 - x^2)$.

(6) Find the sum of the following fractions and simplify your answer.

(a) $\frac{1}{x} + \frac{1}{y}$

- common denominator is xy

$\frac{1}{x} = \frac{y}{xy}$, $\frac{1}{y} = \frac{x}{xy}$

- $\frac{1}{x} + \frac{1}{y} = \frac{y}{xy} + \frac{x}{xy} = \frac{y+x}{xy}$

(b) $\frac{b}{2a} + \frac{1}{b}$

- common denominator is $2ab$

$\frac{b}{2a} = \frac{b^2}{2ab}$

$\frac{1}{b} = \frac{2a}{2ab}$

- $\frac{b}{2a} + \frac{1}{b} = \frac{b^2}{2ab} + \frac{2a}{2ab} = \frac{b^2 + 2a}{2ab}$

(7) Simplify $\frac{x}{\frac{1}{x} + \frac{1}{y}}$.

From (a), $\frac{1}{x} + \frac{1}{y} = \frac{y+x}{xy}$

- $\frac{x}{\frac{y+x}{xy}} = \frac{x}{1} \div \frac{y+x}{xy} = \frac{x}{1} \cdot \frac{xy}{y+x} = \frac{x^2y}{y+x}$

(8) We asked all the students of UCLA what their favorite animal was. $\frac{1}{2}$ of the students said their favorite animals were apes. $\frac{1}{6}$ of the students said that their favorite animals were bats. The remaining 7000 students said their favorite animals were cats.

(a) If a is the number of students who said their favorite animals were apes and x is the number of students at UCLA, write a in terms of x.

$a = \frac{1}{2}x$

(b) If b is the number of students who said their favorite animals were bats and x is the number of students at UCLA, write b in terms of x.

$b = \frac{1}{6}x$

(c) In terms of x, how many students chose apes or bats as their favorite animals?

$a + b = \frac{1}{2}x + \frac{1}{6}x = \frac{2}{3}x$

(d) From you answer in part (c), in terms of x, how many students chose cats as their favorite animals?

$\frac{2}{3}x$ didn't choose cat so $\frac{1}{3}x$ chose cats.

$c = \frac{1}{3}x$

(e) Use the fact that the number of students who like cats is 7000 to find the total number of students.

$C = \frac{1}{3}x = 7{,}000$

$x = 21{,}000$

(9) The sum of three consecutive odd integers is 51. Write an equation where x is the smallest integer of the three and then solve for x.

1st integer: x
2nd integer: $x+2$
3rd integer: $x+4$

$(x)+(x+2)+(x+4) = x+x+2+x+4$

$= 3x+6 = 51$

$3x = 51-6 = 45$

$x = 45 \div 3 = 15$

Fractions and Decimals

(1) Are the following numbers rational? If so, write it in the form $\frac{a}{b}$ where a and b are integers.

(a) $0.\overline{345}$

let $x = 0.\overline{345}$

$1000x = 345.\overline{345}$

$1000x - x = 345.\overline{345} - 0.\overline{345} = 345 = 999x$

$999x = 345$

$x = \frac{345}{999} = \frac{115}{333}$

(b) $\sqrt{2}$

$\sqrt{2}$ is not rational

(c) $0.4\overline{17}$

let $x = 0.4\overline{17}$

$100x = 41.7\overline{17}$

$100x - x = 41.7\overline{17} - 0.4\overline{17} = 41.3 = 99x$

$99x = 41.3$

$x = \frac{413}{99} = \frac{413}{990}$

(d) 0.10110111011110...

Not rational because it is not repeating

(e) 0.123756273

$= \frac{123756273}{1000000000}$

(2) Determine whether the following fractions have repeating or terminating decimal expansions. Remember that terminating fractions can be written in the form $\frac{a}{2^m 5^n}$.

(a) $\frac{1}{7}$

Repeating

(b) $\frac{125}{15}$

$= \frac{25}{3}$ repeating

(c) $\frac{21}{150}$

$= \frac{7}{50} = \frac{7}{2^1 \cdot 5^2}$ terminating

(d) $\frac{1234}{700}$

Repeating

(e) $\frac{77}{7 \cdot 10^n} = \frac{11}{10^n} = \frac{11}{2^n 5^n}$ terminating

Coprimes and Modular Arithmetic

(1) If a and b are coprimes, what is gcd(a,b)?

gcd (a,b) = 1.

(2) Are the following numbers coprimes?

(a) 2, 5

gcd (2,5) = 1 no

(b) 3, 121

gcd (3,121) = 1 no

(c) 7, 15

gcd (7,15) = 1 no

(d) 49, 50

gcd (49,50) = 1 no

Letter	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
Number	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25

(2) Emily sent the following message to Micaela:
"What do you call a pile of kittens? The answer is '0 24 28 8 4 1 13 0 26 1' encrypted using Simplified RSA using the multiplicative encoder 7 in modulo 30."

(a) Find the multiplicative inverse of 7 in modulo 30 to find the multiplicative decoder. Remember that the multiplicative inverse of a number p in modulo m is a number such that $p \cdot q \pmod{m} = 1$.

Find values that are equal to 1 (mod 30)
31 = 1 (mod 30) but we can't find an integer q such that $7 \times q = 31$
61 = 1 mod 30 but $7 \cdot q = 61$ doesn't exist
91 = 1 mod 30. $7 \times 13 = 91 = 1 \pmod{30}$ so the inverse is 13..

(b) Multiply each number in the answer by the multiplicative decoder in modulo 30.

original	0	24	28	8	4	1	13	0	26	1
x13	0	312	364	104	52	13	169	0	338	13
mod 30	0	12	4	14	22	13	19	0	8	13

(c) Translate the number message into a letter message using the cipher key above.

Number	0		12	4	14	22	13	19	0	8	13
Letter	A		M	E	O	W	N	T	A	I	N

Arithmetic and Harmonic Means

(1) Suppose Nicole and Suhani are wrapping presents. Nicole can wrap 4 presents an hour and Suhani can wrap 5 presents per hour.

(a) What is the rate at which Nicole can wrap presents?

4 presents / hour

(b) How many minutes would it take Nicole to wrap one present? flip rate to get time!

$\frac{1 \text{ hour}}{4 \text{ presents}} = \frac{1}{4}$ hour / present = 15 minutes

(c) What is the rate at which Suhani can wrap presents?

5 presents / hour

(d) How many minutes would it take Suhani to wrap one present? flip rate to get time!

$\frac{1 \text{ hour}}{5 \text{ presents}} = \frac{1}{5}$ hour / present = 12 minutes

(e) If Nicole and Suhani work together, how many presents can they wrap in an hour? Is this the arithmetic ~~mean~~ sum or the harmonic ~~mean~~ sum of the rates?

Same amount of time, so arithmetic sum.

(4 + 5) presents / hour = 9 presents / hour.

(f) If Nicole and Suhani wrap the same number of presents, how many presents can they wrap in an hour? Is this the arithmetic ~~mean~~ sum or the harmonic ~~mean~~ sum of the rates?

Same amount of presents, so harmonic sum.

$\frac{1}{4}$ hours for 1 present

$+ \frac{1}{5}$ hours for 1 present

$\frac{9}{20}$ hours for 2 presents

$\frac{9}{40}$ hours / present

$\frac{40}{9}$ presents / hour.