

## NONCLASSICAL CONSTRUCTIONS

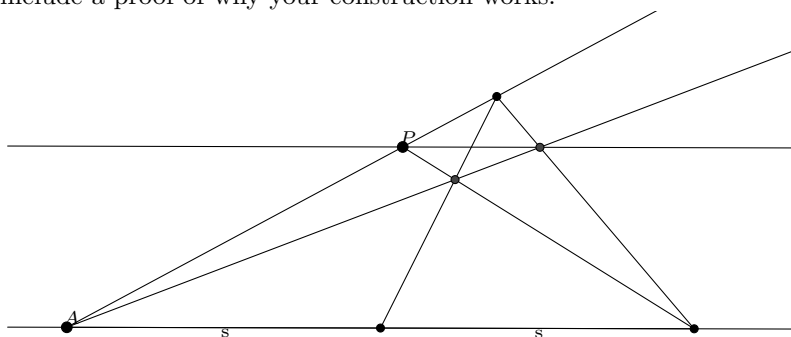
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**Problem 1** (Ceva's Theorem). Let  $ABC$  be a triangle and  $P$  a point inside of it. Let the lines  $AP$ ,  $BP$ , and  $CP$  intersect the sides of the triangle in the points  $X$ ,  $Y$ , and  $Z$ . Show that

$$\frac{\overline{AZ}}{\overline{ZB}} \frac{\overline{BX}}{\overline{XC}} \frac{\overline{CY}}{\overline{YA}} = 1.$$

*Hint:* Draw parallels to the triangle sides through  $P$  and try using the intercept theorem.

**Problem 2** (Construction of parallels through a given point,\*). Use the following sketch to construct the parallel through a point  $P$  to a given line through the point  $A$  using only a marked ruler. Be sure to include a proof of why your construction works.



**Problem 3** (Transfer of line segments,\*). Use the previous constructions and the interception theorems to construct a point  $Y$  on a given line containing a point  $B$  with  $\overline{BY} = \overline{AX}$  where the points  $A$  and  $X$  are given. Be mindful to distinguish three different cases.

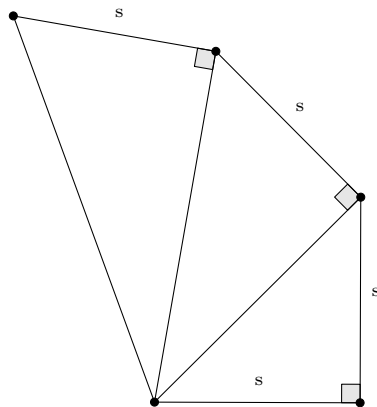
**Problem 4.** Construct the midpoint of a given line segment and then construct half of a given angle.  
*Hint:* Use the intercept theorem.

**Problem 5** (Construction of perpendiculars.) (i) Construct a line perpendicular to a given line  $g$ .

*Hint:* Choose an arbitrary line  $h$  intersecting  $g$  and use marked line segments to get a rectangle with diagonals  $g$  and  $h$ . Now rotate the rectangle by  $90^\circ$ .

(ii) Can you use this to transfer angles?

**Problem 6.** Determine any angles in the following sketch. What does this make possible to construct



with a marked ruler?

**Problem 7.** Construct an equilateral triangle with a marked ruler using as few steps as possible.