

Well-definition of addition modulo n

Prove each of the following important statements about addition modulo n , where n is any integer greater than or equal to 2. When applicable, explain how it has changed your life.

1. If $a \equiv 1 \pmod{3}$, then $a + 1 \equiv 2 \pmod{3}$.
 2. If $a \equiv 2 \pmod{3}$, then $a + 5 \equiv 1 \pmod{3}$.
 3. If $a \equiv 3 \pmod{4}$, then $a + 1 \equiv 0 \pmod{4}$.
 4. If $3a \equiv 2 \pmod{7}$ and $2a \equiv 6 \pmod{7}$, then $a \equiv 3 \pmod{7}$.
 5. If $a \equiv b \pmod{n}$, then $a + 1 \equiv b + 1 \pmod{n}$.
 6. If $a \equiv b \pmod{n}$, then for any integer k , $a + k \equiv b + k \pmod{n}$.
- ∞ . If $a_1 \equiv b_1 \pmod{n}$ and $a_2 \equiv b_2 \pmod{n}$, then $a_1 + a_2 \equiv b_1 + b_2 \pmod{n}$.

The last statement (problem “ ∞ ”) is called the well-definition of addition modulo n .

Well-definition of multiplication modulo n

Prove each of the following important statements about multiplication modulo n , where n is any integer greater than or equal to 2. When applicable, explain how it has changed your life.

1. If $a \equiv 0 \pmod{7}$ then $5a \equiv 0 \pmod{7}$
 2. If $a \equiv 2 \pmod{3}$ then $3a \equiv 6 \pmod{3}$.
 3. If $a \equiv 4 \pmod{10}$, then $12a \equiv 8 \pmod{10}$.
 4. If $a^2 \equiv 4 \pmod{7}$, then $a \equiv 2 \pmod{7}$.
 5. If $a \equiv b \pmod{n}$, then $5a \equiv 5b \pmod{n}$.
 6. If $a \equiv b \pmod{n}$, then for any integer k , $ak \equiv bk \pmod{n}$.
- ∞ . If $a_1 \equiv b_1 \pmod{n}$ and $a_2 \equiv b_2 \pmod{n}$, then $a_1a_2 \equiv b_1b_2 \pmod{n}$.

The last statement (problem “ ∞ ”) is called the well-definition of multiplication modulo n .