Some More Linear Congruences

For each of the congruences below, list all possible values of x modulo m, and check which values give solutions to the congruence.

1. $2x \equiv 1 \pmod{3}$
2. $3x \equiv 7 \pmod{2}$
3. $2x \equiv 2 \pmod{4}$
4. $4x \equiv 3 \pmod{6}$
5. $4x \equiv 2 \pmod{6}$
6. $4x \equiv 4 \pmod{6}$

Some Facts to Prove

Prove each of the statements below:

1. If $3x \equiv 1 \pmod 4$, then $x \equiv 3 \pmod 4$.
2. If $x \equiv 2 \pmod 3$, then x is not divisible by 6.
3. If $x \equiv 4 \pmod{12}$, then x is not a prime number.
4. If x ends in a 0, then $x \equiv 0$, 5, 10, 15, or 20 $\pmod{25}$.
5. If $x \equiv 2 \pmod 6$ and $x \equiv 3 \pmod 5$, then $x \equiv 8 \pmod{30}$.
6. The smallest positive integer x which is congruent to 4 $\pmod{100}$, and congruent to 2 $\pmod 7$, is 604.
7. In the previous problem, there is no largest such positive integer x.