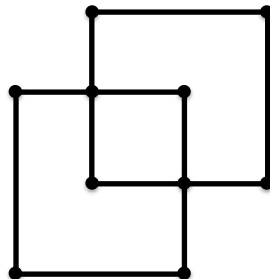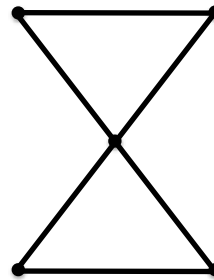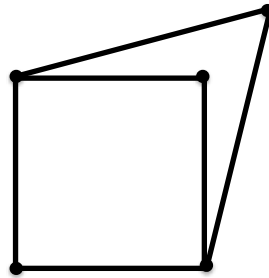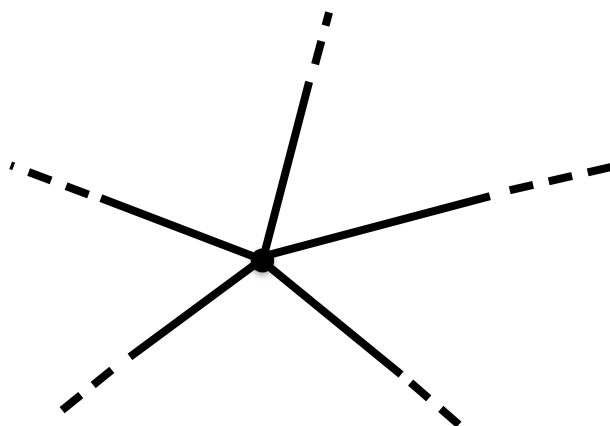# End of Year Review

June 1, 2014

Graphs

1. Cory takes a trip to Paris, France. He goes to see different attractions, such as museums, restaraunts, parks, etc. Each of the vertices in the graphs below represent different attractions. Each of the edges in the graphs represent different streets. Cory wants to start at a certain attraction in the morning, visit all the attractions, and end back at the starting location. He does not want to walk on the same street twice, but he does not mind visiting the same attraction more than once. Circle the graphs below where it is possible for Cory to do this.

2. Cory buys a map in Paris, and brings it back home so that he can show Emmanuelle where he visited. Unfortunately, on his way back from Paris, someone spilled coffee on the map, leaving only a small piece of the map legibile. Below is the portion of the map that is legible. Cory showed Emmanuelle this portion of the map, and told her that he was able leave his hotel, see all the attractions, and get back to his hotel by traveling on each street exactly once. Is Cory telling the truth? Explain.

3. Draw a graph that has the following properties:

   (a) There are a total of 9 vertices.
   (b) There are only 2 circuits, and they do not share common vertices.
   (c) One circuit contains 5 vertices.
   (d) The other circuit contains 4 vertices.
   (e) There are two odd vertices.

Modular Arithmetic

4. Reduce the numbers in modular arithmetic.

   (a) $12 \equiv \qquad (\text{mod } 5)$

   (b) $18 \equiv \qquad (\text{mod } 8)$

   (c) $80298391 \equiv \qquad (\text{mod } 2)$

   (d) $-10 \equiv \qquad (\text{mod } 4)$

   (e) $-1 \equiv \qquad (\text{mod } 6)$

   (f) $-1 \equiv \qquad (\text{mod n})$

   (g) $n + 5 \equiv \qquad (\text{mod n}) \text{ where } n > 5$

   (h) $10n + 3 \equiv \qquad (\text{mod n}) \text{ where } n > 3$

5. Reduce the following expressions in modular arithmetic.

   (a) $6 + 8 + 4 + 100 \equiv \qquad (\text{mod } 5)$

   (b) $8 \times 6 \times 4 \times 11 \equiv \qquad (\text{mod } 3)$

6. We will use the number 8142 to explain a divisibility test for dividing by 9.

   (a) Write the expanded form of 8142.

   $8142 =$

   (b) Reduce the following numbers in mod 9.

   $1 \equiv \qquad (\mathrm{mod}\ 9)$

   $10 \equiv \qquad (\mathrm{mod}\ 9)$

   $100 \equiv \qquad (\mathrm{mod}\ 9)$

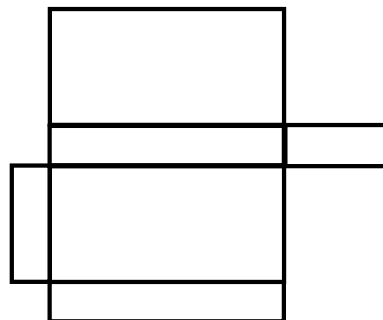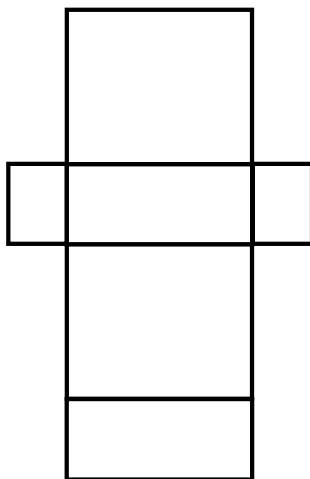 $1000 \equiv \qquad (\mathrm{mod}\ 9)$

   (c) Using yours answers from part (a) and part (b), explain a divisibility test for dividing by 9.
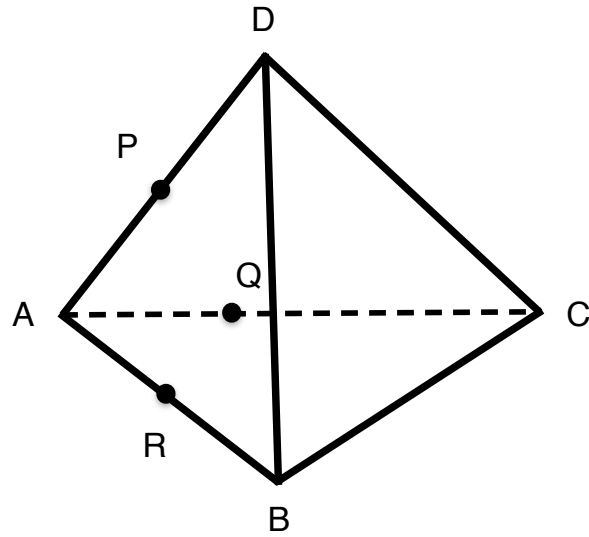
Geometry

7. What are the possible intersections of two lines in 3 dimensions? Give an example of each possibility.

8. What are the possible intersections of two planes? Draw each possibility.
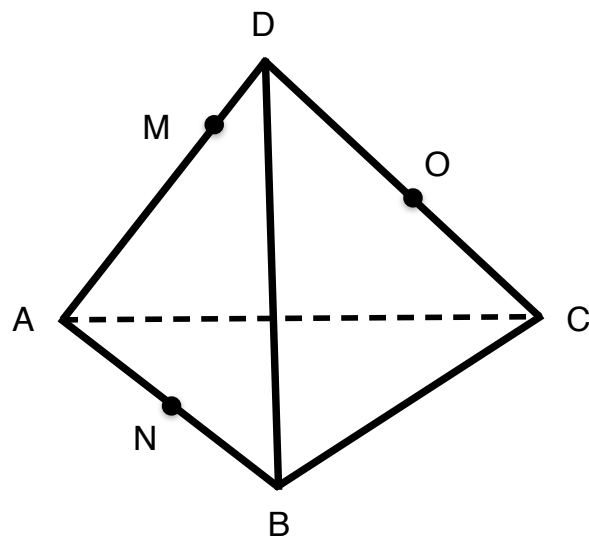
9. Indicate which of the following 2D shapes can be folded into rectangular prisms.

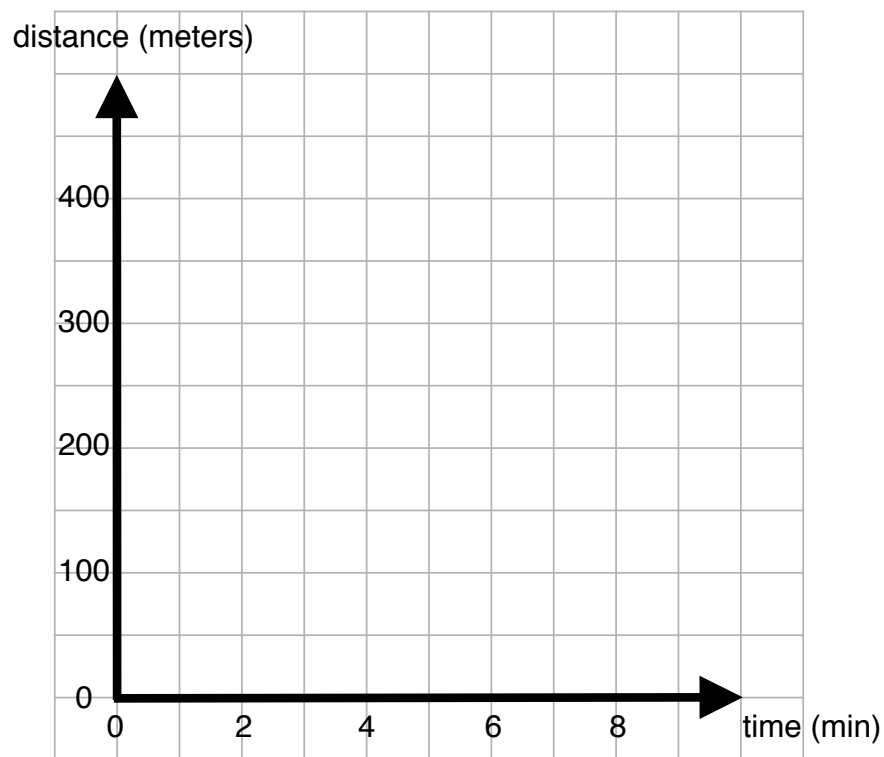10. Draw the cross-section of the tetrahedra $DABC$ going through the points PQR. Shade in the cross-section.

11. Draw the cross-section of the tetrahedra $DABC$ going through the points MNO. Shade in the cross-section. (*Hint*: Start by extending AC and finding intersection of AC and MO).

Rescaling

12. Kaley cuts a rectangular piece of paper with area $45\ cm^2$ into smaller rectangles that have dimensions $\frac{1}{3}$ as large as the original rectangle. What is the area of a smaller rectangle?

13. 100 bricks were used to build a wall. Each brick weighs $1\ lb$. If 100 bricks are used that have dimensions twice as big as the original bricks, how much will this larger wall weigh?

The Distance Formula

14. Lucy travels at speed v for time t. Write down how far Lucy travels (S) during this time period.

$S =$

15. Kaley and Emmanuelle start from the same point in opposite directions. Kaley walks at a speed of 60 meters per minute, and Emmanuelle walks at a speed of 50 meters per minute. How far apart will they be in 6 minutes?

16. Lucy is at home, and walks to the store to get coffee. The store is 400 meters from Lucy's house, and it takes her 4 minutes to get there. She stays at the store for 2 minutes. Then, she takes 3 minutes to walk back to her house. Graph how Lucy's distance from her house changes wtih time.

What is the speed at which Lucy travels from her house to the store?

Cryptarithms

17. Solve the following cryptarithm. Remember that each letter represents a distinct digit. Different letters cannot represent the same digit, and the same letters cannot represent different digits.

$$
\begin{array}{r}
S\ E\ E \\
-\quad A\ S \\
\hline
A\ S
\end{array}
$$