

# Scaling Areas and Volumes

February 23, 2014

## Scaling with Areas

1. A square has side length 5 cm. What is the area of the square?
  
  
  
  
  
  
  
  
  
  
2. What is the area of a square that has side length  $L$ ?
  
  
  
  
  
  
  
  
  
  
3. It takes 3 ounces of ink to print a large picture on a poster.
  - (a) How many ounces of ink will it take to print the picture if its dimensions are twice as large?
  
  
  
  
  
  
  
  
  
  
  - (b) What if the dimensions are three times as large?

4. Richmond cuts a square paper into smaller squares that have a side length that is half of the original side length. How many squares did he make?

5. Andrew has a large number of small, square pieces of paper. Using these squares, he makes a larger square whose side length is three times larger than the original squares' lengths.

(a) How many squares does he need to do this?

(b) How many times larger is the area of the larger square than the area of the smaller square?

(c) If the side length of the smaller square is 2 cm, what is the area of the larger square?

6. Kate has a square piece of paper that has a side length of 10 cm. She cuts the square into smaller squares of side length 1 cm. How many total squares did she make?

7. Jason makes a picture out of square tiles of size  $1 \times 1$ . Prakash makes a picture of the same design out of tiles of size  $2 \times 2$ . Elliot uses tiles of size  $3 \times 3$  to make a picture using the same design. Let  $A_1$  be the area of Jason's picture,  $A_2$  be the area of Prakash's picture, and  $A_3$  be the area of Elliot's picture.

(a) How many times larger is the area of Prakash's square when compared to Jason's square?

(b) Express  $A_2$  in terms of  $A_1$  (fill in the blank).

$$A_2 = \text{---} \times A_1$$

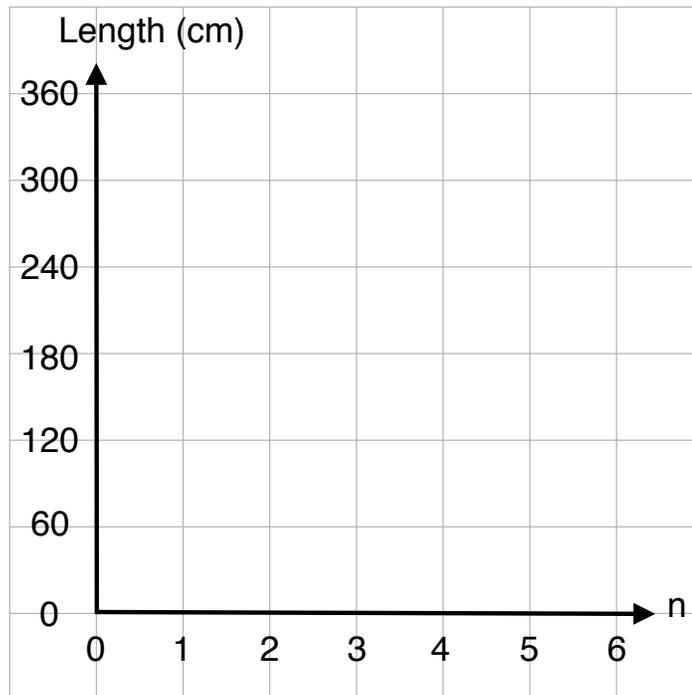
(c) Express  $A_3$  in terms of  $A_1$ .

8. In this problem, Kara will cut a large square into smaller squares, and then will make a long strip out of the smaller squares. Kara will also find out if we can make this strip as long as she would like by making sufficiently many squares. The large square has side length 60 cm.

(a) Fill out the table below. Let  $n$  be the number of strips made when the paper is cut along one side of the large square.

s	n	Number of small squares	Length of Strip
30			
20			
10			
...	...	...	...
$60/n$			

(b) Sketch a graph showing the total length of the strip as a function of  $n$ .



(c) Can you make the length of the strip as big as you like?

(d) Into how many small squares should Kara cut the large square in order to be able to form a strip of length 3600 cm?

### Scaling with Volumes

1. A cube has a length of 3 cm. What is the volume of the cube?

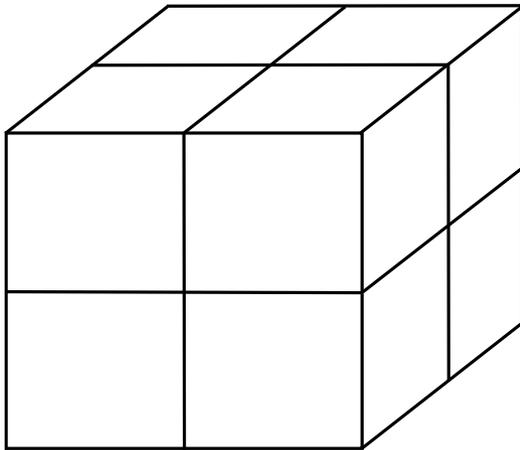
2. What is the volume of a cube that has length  $L$ ?

3. A set of dolls consists of two dolls, a mother and a daughter, which have the same shape. The daughter doll weighs 1 lb.

(a) If the mother's dimensions are twice as big as those of the daughter, how much does the mother doll weigh?

(b) If the mother's dimensions are three times as big as those of the daughter, how much does the mother doll weigh?

4. Lev has a large cube. He cuts the cube into smaller cubes such that the side length of the smaller cubes is half of the side length of the large cube. How many cubes did he make? Use the cube below to help you solve the problem.



5. Jereth has a large number of small cubes. He wants to make a larger cube that has a side length three times larger than the small cubes.

(a) How many cubes does he need to do this?

(b) How many times greater is the volume of the larger cube than a smaller cube?

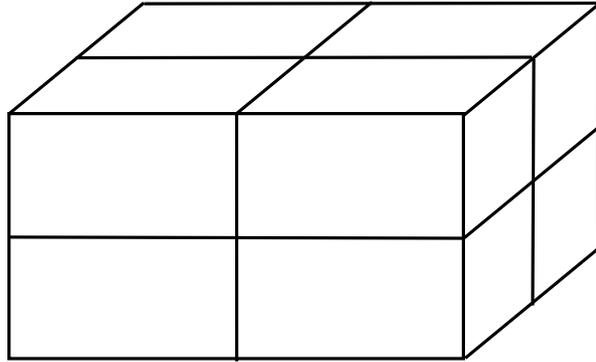
(c) If the volume of a small cube is  $2 \text{ cm}^3$ , what is the volume of the larger cube

6. Jason makes a sculpture out of cubes with side length 1 cm. Prakash makes a sculpture of the same design out of cubes with side length 2 cm. Elliot uses cubes of side length 3 cm to make a sculpture of the same design. Let  $V_1$  be the volume of Jason's picture,  $V_2$  be the area of Prakash's picture, and  $V_3$  be the area of Elliot's picture.

(a) Express  $V_2$  in terms of  $V_1$ .

(b) Express  $V_3$  in terms of  $V_1$ .

7. Last century, many people used rectangular soap bars to do laundry. After doing laundry seven times, the height, length and width of a soap bar all decreased by a factor of two. Use the bar of soap below to help you answer the questions.

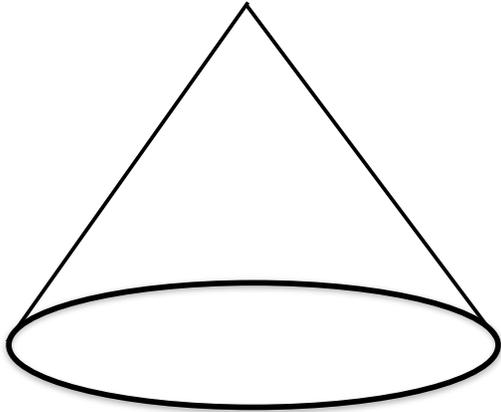


- (a) How many left-over soap bars can you fit into the original soap bar?
- (b) How many more laundries can you do with the remainder of the soap bar?

### More Problems

1. The surface area of a balloon is increased by a factor of 4.
- (a) How many times larger is the radius?
- (b) How many times larger is the volume?

2. A regular party hat weights 100 grams. A giant party hat has dimensions twice as large as the regular party hat. How much does the giant party hat weigh? A party hat has the shape of a cone, which is shown below.



3. A standard birthday cake weighs 5 lb. A large birthday cake has dimensions twice as large as the standard birthday cake. How much does the large birthday cake weigh?

4. What is the difference between the party hat problem and the birthday cake problem?