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**Problem 1** *Give the definition of probability of a random event.*

**Problem 2** *You roll two dice. List pairs of all the possible outcomes in the table below.*


**Problem 3** *You roll two dice. What is the probability of scoring more than eight?*

$$P(\text{the score is } > 8) =$$

**Problem 4** *A wooden cube is painted and cut into 512 smaller cubes of equal size. Then the little cubes are carefully mixed in a non-transparent bag. Once the mixing is finished, you pull out one of them. What is the chance that the cube you pull out has no paint on it?*

**Problem 5** *You toss a coin 9 times. How likely are you to get at least one head?*

**Problem 6** *Use factorials to compute the following numbers.*

a.  $\binom{10}{0} =$

b.  $\binom{10}{8} =$

**Problem 7** Use Pascal's triangle to find  $\binom{7}{5}$ .

**Problem 8** Simplify the following expression.

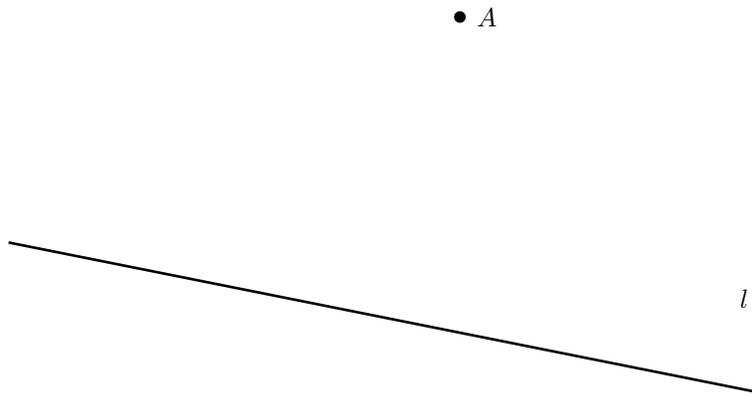
$$(x + y)^3 - \sum_{k=0}^3 \binom{3}{k} x^{3-k} y^k =$$

**Problem 9** *You toss a fair coin 20 times. What is the chance that you get 4 tails?*

**Problem 10** *There are two plates on the table. One plate has 12 identical candies, the other has 6 different fruits. How many ways are there to choose five candies and four fruits? (The treat will be put in a box, so the order does not matter.)*

**Problem 11** *Formulate and prove the Pythagoras' Theorem.  
Use the Claim/Reason chart.*

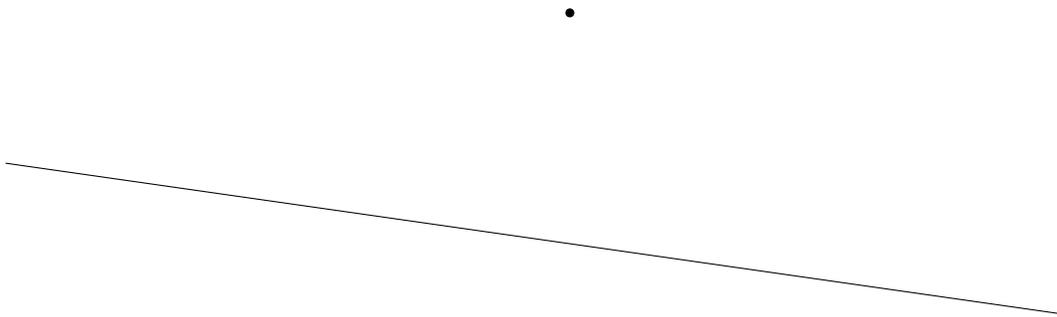
**Problem 12** *Using a compass and a ruler, construct the shortest possible path from the point  $A$  to the straight line  $l$  below.*



*Continues on the next page.*

*Use the Claim/Reason chart to prove that the path you have constructed is indeed the shortest.*

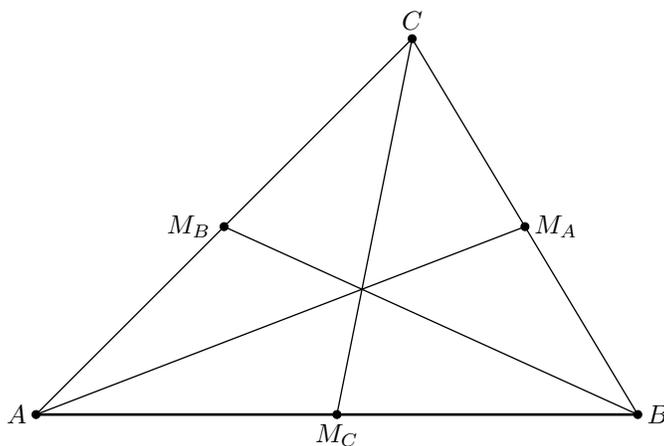
**Problem 13** *Use a compass and a ruler to draw a straight line parallel to the given one and passing through the given point below.*



**Problem 14** *A boy weighing 60 lbs wants to lift his 180 lbs father using a 4-foot-long stick as a lever. Where should he place the fulcrum? Hint: a picture will help.*

**Problem 15** Find barycentric coordinates of the point  $x = 3.2$ .  
*Hint: a picture will help.*

**Problem 16** Prove that the medians of a triangle intersect at one point that divides each median in the ratio  $2 : 1$  counting from the corresponding vertex.



Use the next page to make the Claim/Reason chart.



**Problem 17** *Formulate a statement similar to that in Problem 16 for a triangular pyramid, called a simplex in the grown-up math, in a ten-dimensional space. Draw a 2D projection of the 10D simplex.*