Egyptian Multiplication

April 19, 2013

Ancient Egyptians had an interesting method for multiplying two numbers. Suppose that you have to multiply two numbers (e.g., 21×18). The basic operation for them was multiplying a number by 2. (In other words, doubling). All multiplication problems were reduced by this concept. Here is how they would start multiplying 21×18 (in modern notation):

21	18
1	18
2	36
4	72
8	144
16	288

Here is what they did to complete the multilpication:

1. Below the first number (in this case, 21), they would write all of the **powers of 2** that are smaller or equal to the first number (in this case 21).

2. In the second column, the number is doubled, starting with itself (in this case, 18).

3. After that, the first number would be represented as the sum of the powers of 2 (so that each of the powers of 2 is used at most once). For example, if the first number is 21, they would find $21 = 16 + 4 + 1$.
Using the power of 2 that were used for the sum of the left number, now add the corresponding right column numbers to find the overall product of the multiplication problem:

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              1    8
              7    2
  +    2      8    8
      ——   ——   ——
       3    7    8
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Thus, the solution of the multiplication problem $21 \times 18 = 378$.

Multiply 13 by 22 using Egyptian multiplication.

The goal of our session today is to understand how and why the Egyptian's method of multiplication worked.

1. Would the following sum of powers of 2 be used to represet the value 38 when using Egyptian multiplication?

$$38 = 2 + 4 + 16 + 16$$

   (a) Yes or No. (Circle one.)

   (b) Why?

   (c) Correct the problem.

2. Represent each of the numbers below as a sum of powers of 2, using each power of 2 only once.

   (a) $9 =$

   (b) $14 =$

   (c) $23 =$

   (d) $44 =$

3. Multiply the following numbers using Egyptian Multiplication:

  (a) 13×41

  (b) 41×13

4. Understanding that $a \times b$ is the same as $b \times a$ give the same product for all integers, which integer would you use as the first number in Egyptian Multiplication, the smaller number or the larger number? Why? (i.e with 17 and 35, which one is easier to find the power of 2s for?)

5. Give your own explanation of why Egyptian Multiplication works.

6. In your opinion, is it easier to use the regular multiplication strategy or Egyptian Multiplication?

7. With a partner, have a race to see who can multiply numbers faster. One of you must use Egyptian Multiplication and the other must use regular, long multiplication. Race 4 times switching the type of multiplication you do. Show your work below:

(a) $25 \times 31 =$
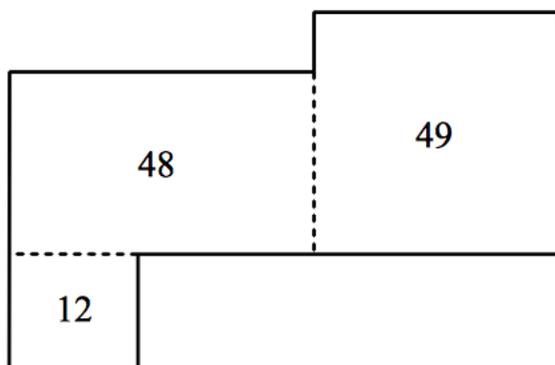
(b) $38 \times 45 =$

(c) $12 \times 63 =$

(d) $17 \times 52 =$

8. Below are shapes that are divided into different rectangles. The area for each rectangle is given. Your job is to label the lengths of all the sides as well as give the total perimeter.

(a)

10

14

Perimeter =

(b)

49

48

12

Perimeter =