A LEVER PLAYING FIELD 2: NEVER SAY LEVER

The Law of the Lever, first proved by the Greek mathematician Archimedes, describes the situation in which a scale/balance/lever will be balanced.

Consider a lever that has a weight of W_1, at a distance D_1 to the left of the fulcrum, and a weight of W_2 at distance D_2 to the right of the fulcrum (as pictured). Then the Law of the Lever states that the two weights will be balanced if, and only if

$$W_1 D_1 = W_2 D_2.$$

More generally, if there are multiple weights on each side, then the *sum* of the weight×distance values on the left side, must equal the sum of the weight×distance values on the right side. For example if there were weights W_1 and W_2 on the left side at distances D_1 and D_2 respectively; and weight W_3 on the right side at distance D_3, then the scale will be balanced if and only if

$$W_1 D_1 + W_2 D_2 = W_3 D_3.$$

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(1) You have a 3-lb. weight and a 5-lb. weight. On the lever below, place the weights so that the scale will be balanced. **In this and all subsequent problems, label the weights you use (here 3 lb, 5 lb), and indicate the distance of each one from the fulcrum.** (Hint: There is not just one possible answer, although there is arguably one that is "simplest".)

Solution: The simplest solution is to place the 5 lb weight 3 ft. from the fulcrum, and the 3 lb weight 5 ft. from the fulcrum. Of course here "feet" can be replaced by any unit of distance; and in fact (which is another way of saying the same thing) we can place them anywhere on the scale so long as the ratio of 5lb distance to 3lb distance is 3:5.

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(2) You have a 1-lb. weight, a 2-lb. weight, and a 3-lb. weight. Place them on the scale so that it will be balanced, being sure to label the weights and their distances. (Again, there are multiple possible answers.)

Solution: For instance, place the 1 lb weight and 2 lb weight at the same location on one side, and the 3 lb weight an equal distance on the other side.

(3) Place a 10-lb. weight on the following scale, so that it balances with a weight which is 15 ft. from the fulcrum on the right side. You choose the weight on the right side, and the distance of the 10-lb. weight.

Solution: For instance, place the 10 lb weight 15 ft. from the fulcrum, and use another 10lb weight on the right side. (!)

(4) Now solve problem (3) again, but give a different answer:

Solution: For instance, place the 10 lb. weight 30 ft. from the fulcrum, and use a 20 lb weight on the right side.

(5) This scale will use a 3kg weight, a 7kg weight, and a 9kg weight. The distances of the weights should be 2m, 3m, and 3m (but not necessarily in that order, and I'm not telling you on which side of the balance each distance should be!) Can you figure out where to place the weights, according to those rules, to balance the scale?

Solution: Place the 3kg weight on the left side 2m away, the 7kg also on the left 3m away, and the 9kg on the right side 3m away. (Or reverse left and right.)

(6) The following balance is not balanced. It currently has a weight of 2kg suspended 1.6m to the left of the fulcrum, and a weight of 1kg suspended 6.4m to the right. **(Draw and label these weights and distances.)** The total length of the bar is 20m (10 on the left, 10 on the right). What is the weight of the smallest possible weight you could add to balance the scale, and where should you place it?

Solution: Currently the torque on the left side is 3.2 kg-m, and on the right side is 6.4 kg-m. In order to balance, we must add 3.2 kg-m of torque to the left side, and the way to do this with the least possible weight is to place it at the far left end of the bar, 10m from the fulcrum. In this case, it needs to be .32 kg, since (.32 kg)(10 m) = 3.2 kg-m.

(7) The following bar has a weight of 15 lbs. at the far left and and 3 lbs. at the far right (draw them!). Where along the bar should you place the fulcrum so that it will balance? (Draw and label!) You can imagine that the bar itself weighs nothing.

Solution: We're not given the length of the bar, so our answer will be some fraction of the bar's overall length. With that in mind, let's solve the problem assuming a bar of length 1. Call the location of the fulcrum x (in units from the left endpoint of the bar). Then we need $15 \cdot x = 3 \cdot (1 - x)$, or $15x = 3 - 3x$, or $18x = 3$, or $x = 1/6$. Thus the fulcrum should be placed 1/6 of the way along the bar, dividing it into segments which are in a ratio of 1:5.
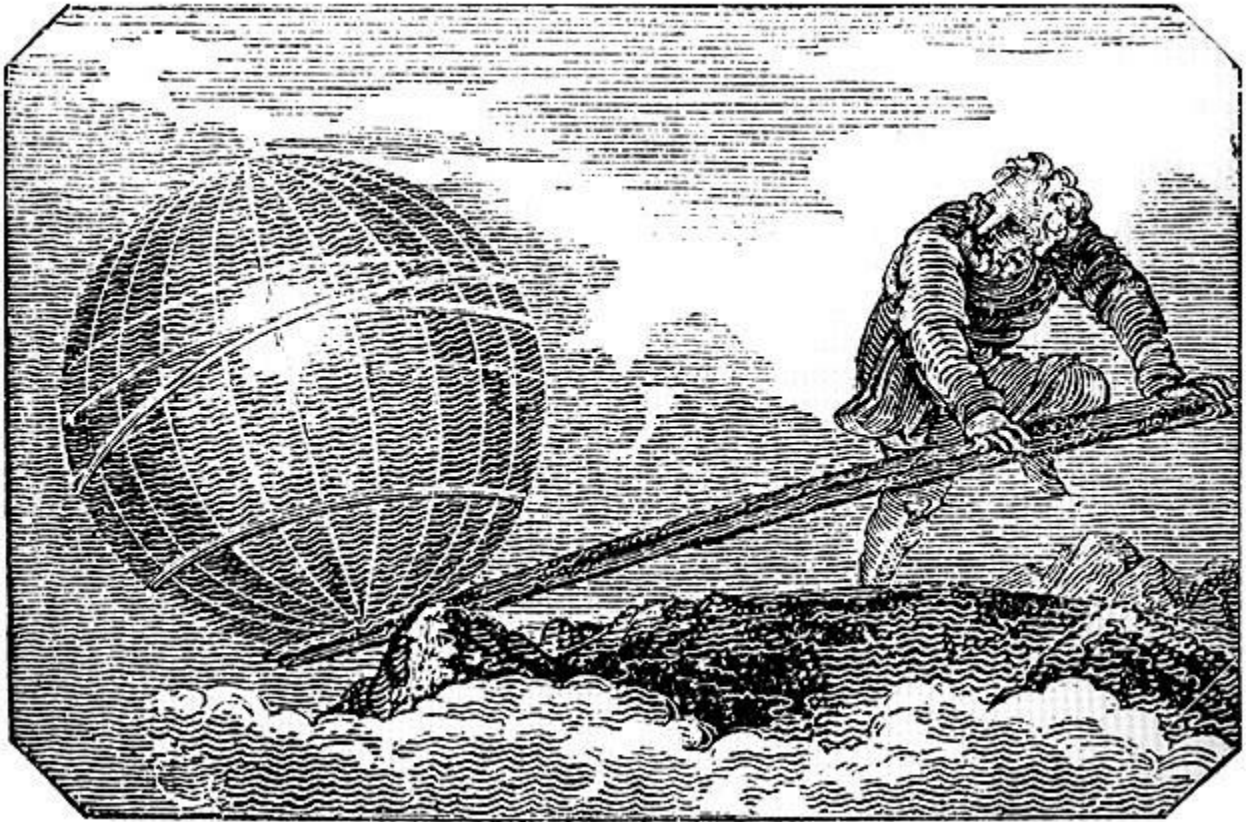
(8) In the previous problem you were supposed to imagine that the bar weighed nothing. Of course in reality a bar WILL weigh something. The bar below is actually rather heavy; it weighs 4 lbs. The distance from the fulcrum to the left end of the bar is 2 feet, and I have hung a 1 lb. weight at the left end. There is nothing hanging on the right side.

How long is the bar? (Picture may not be to scale.) (Hint: Where is a bar's center of gravity, in general?)

Solution: The bar's center of gravity is (of course) at the very center of the bar. Let's have x be a variable describing the total length of the bar (which is what we are looking for). Then the distance from the *center* of the bar (which is its center of gravity) to the fulcrum is exactly $(x/2) - 2$. Since we can think of the weight of the bar as being concentrated at its center of gravity, then applying the law of the lever gives us an equation $1 \cdot 2 = 4 \cdot (x/2 - 2)$, or $2 = 2x - 8$, or $2x = 10$, or $x = 5$. So the total length of the bar is 5 feet. (If we check the answer, we see that the midpoint of the bar is 2.5 feet from the left end, so the midpoint is .5 feet to the right of the fulcrum, and indeed $1 \cdot 2 = 4 \cdot (.5)$, so our answer checks out.)

(9) Archimedes reportedly once boasted about the usefulness of levers, saying, "Give me the place to stand, and I shall move the earth." (See picture—not to scale.)

(a) About how much mass ("weight") does the Earth have, in either lbs. or kilograms? (Use any knowledge you, your neighbors, your assistant, etc. have to formulate a best-guess estimate.)

Solution: I happen to recall that the circumference of the world is $C \approx 25,000$ mi., which means according to the relationship $C = 2\pi r$ its radius is $r \approx 4,000$ mi. Then using $V = 4/3\pi r^3$, the volume of the world is $V \approx .4(4000)(4000)(4000)\ mi^3 = 25600000000\ mi^3 \approx 2.5 \times 10^{10}\ mi^3$. There are $\approx 5,000$ feet in a mile, so that volume is about $2.5 \times 10^{10} \times 5000^3\ ft^3 = 2.5 \times 5^3 \times 10^{10} \times 10^9\ ft^3 \approx 3,000 \times 10^{19}\ ft^3 = 3 \times 10^{22} ft^3$. Now, my best guess, but something of a wild guess, is that a cubic foot of earth weighs roughly 50lb. on average. (It should weigh more as you get closer to the center of the earth and matter is compressed more.) Thus the total weight of the earth is $\approx 150 \times 3 \times 10^{22}\ lbs = 4.5 \times 10^{24}\ lbs$. (Update: After checking the Internet, it appears I underestimated by a factor of 3. Not too bad!)

(b) About how many lbs. or kilograms (use same as above) of downward force can a human generate? (Use any knowledge you, your neighbors, your assistant, etc. have to formulate a best-guess estimate.)

Solution: A human, unaided, can exert at most their own weight in downward force, so if we imagine Archimedes was a large-ish man (I have no idea his actual stature), perhaps 200lbs.

(c) Given your answers to (a) and (b), about how long a lever would Archimedes need in order to move (or just balance) the world, assuming he found a place to stand, and

the fulcrum were placed 1 meter from where the world rested on the lever? (You can imagine the lever itself is weightless, like in Problem (7).)

Solution: According to the law of the lever, we would need $(4.5 \times 10^{24}\ lbs) \times (1m) = (200\ lbs) \cdot ((length - 1)\ m)$. The "-1" after length here is irrelevant since it is vastly outweighed by the scale and approximation error of everything else in the problem, so my best guess is a lever of length $4.5 \times 10^{24}/200 \approx 2 \times 10^{22}m$. (The diameter of the observable universe is roughly $9 \times 10^{26}m$ across, for comparison!) (Update: Using the more accurate figure for the weight of the Earth would give $6 \times 10^{22}m$ as the answer. Or it would give the same answer, if we additionally assume that Archimedes actually weighed 600 lbs instead of 200 lbs!).