THE TRIANGLE INEQUALITY II

MATH CIRCLE (INTERMEDIATE) 2/5/2012

1) Prove that the distance between any two points inside a triangle is not greater than half the perimeter of the triangle.

2) Given points A and B are on one side of line l. We want to draw a path from A to B that intersects l. Find the shortest such a path.

3) Point A, inside an acute angle, is reflected in either side of the angle to obtain points B and C. Line segment BC intersects the sides of the angle at D and E (i.e. E is close to C than B). Show that:

a) $EC = EA$ and $BD = DA$.

b) $DE < BD + EC$.

c) $BC/2 > DE$.

4) A fly sits on one vertex of a wooden cube. What is the shortest path it can follow to the opposite vertex?

5) If point O is inside triangle ABC, prove that $AO + OC < AB + BC$.

6) Point C lies inside a given right angle, and points A and B lie on its sides. Prove that the perimeter of triangle ABC is not less than twice the distance OC, where O is the vertex of the given right angle.

Challenge 1) Prove that the length of median AM in triangle ABC is not greater than half the sum of sides AB and AC.

Challenge 2) Prove that in any triangle, the sum of the lengths of the three medians is not greater than the triangle's perimeter.

Problems are taken from:

- D. Fomin, S. Genkin, I. Itenberg "Mathematical Circles (Russian Experience)"
- Previous UCLA Math Circle notes

Warm up 1) Do there exist natural numbers a and b such that $ab(a - b) = 65065$?

Warm up 2) Find a point inside a convex quadrilateral such that the sum of the distances from the point to the vertices is minimal.