1. Measurement Errors

Problem 1. (3 points each sub-problem, and 1 additional point for solving all)

(1) Let $y = e^x$. Use the fact that $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$ to find $\Delta(y)$ in terms of x and $\Delta(x)$. Make sure to simplify your answer as much as possible.

(2) Now suppose $z = e^{ax}$. Find $\Delta(z)$ in terms of x and $\Delta(x)$. It may help to first find it in terms of ax and $\Delta(ax)$.

(3) The spread of a virus can often be modeled by exponential growth (at least at the very beginning). The number of infected people on day t, $y(t)$, is represented by the equation $y(t) = e^{at}$, where $a = \frac{\ln 2}{\tau}$ and τ is the time it takes for the number of infected people to double. Suppose the number of infected people doubles every 6 days. Find the error $\Delta(y)$ on day 18.

Problem 2. (10 points) You have a spherical non-water balloon full of water. Unfortunately, you're not allowed to keep it. Instead, you must empty it into a cubic box and go to bed, *without dinner*. I forgot to mention, you have so many cubes of all sizes. Being the space-considerate person that you are, you want to choose the smallest cube that will hold **all** the water from the non-water balloon. You measure the diameter of the non-water balloon to be 11 inches. Use the measurement error of the volume to find the side length of the cube. (`Make sure that no water is spilled`.)

Problem 3. (10 points) You and your buddy Buddy are so excited to approximate π. His son Sonny comes up with the best idea some side of the Mississippi. You will construct a perfect circle. Nature does it all the time, right? How hard can it be? Buddy will measure the diameter of the circle. Sonny's pal Paul will take a string and wrap it around the circle to measure the circumference. Being a good Math Circle student, you know how to get π given the diameter and the circumference of a circle. Buddy measures the diameter to be 45 feet and Paul measures the circumference to be 138 feet. Use the concept of measurement errors to explain why at least one of Sonny's dad and friend is terrible at measuring things.

Problem 4. (5 points) The Mars Climate Orbiter (formerly the Mars Surveyor '98 Orbiter) was a 638-kilogram (1,407 lb) robotic space probe launched by NASA on December 11, 1998 to study the Martian climate, Martian atmosphere, and surface changes and to act as the communications relay in the Mars Surveyor '98 program for Mars Polar Lander. However, on September 23, 1999, communication with the spacecraft was permanently lost as it went into orbital insertion. The spacecraft encountered Mars on a trajectory that brought it too close to the planet, and it was either destroyed in the atmosphere or escaped the planet's vicinity and entered an orbit around the Sun. An investigation attributed the failure to a measurement mismatch between two software systems: metric units by NASA and non-metric (imperial or "English") units by spacecraft builder Lockheed Martin. Now that's what I call a *measurement error*!

Rate this joke from 0 to 5, 0 being zero and 5 being five.