Winter Quarter Game 2

Nikita

1 Logics

1. Irene has 100 coins, some of them are dimes, and some are quarters. It is known that at least one coin is a dime, and at least one of any two coins is a quarter. How much money does Irene have?

2. Kolya, Borya, Vova and Yura took the first four places in the competition, and no two of them shared a place. When asked who took which place, Kolya replied: "Neither the first nor the fourth"; Borya said: "Second" and Vova noticed that he was not the last. Everyone told the truth. Which place did each of the boys take?

3. At the school disco, Albert, Boris, Chen and Dean, all from different classes, danced with the girls, but each danced not with his classmate. Elena danced with Albert, Fatima with Giselle's classmate, Boris with Chen's classmate, and Chen with Hangjie. Who is in the same class with whom?

4. To the question: "How old are you?" the Martians looked at each other. One of them smiled and said: "Mi recently turned 22 months old, but Mu is older, she is 21 million years old." The second also smiled: "In fact, Mi is 21 million years old, but Mo is only 19 thousand years old." And here the third laughed: "Mo is really only 18 weeks old, 21 million years is actually Ma." And the fourth honestly said that there were no peers among them, and in each answer the age of only one of the Martians was stated correctly. Determine the age of each.

5. On the island of knights and knaves and spies one day 10 natives are asked what time it was. Six answered "one in the afternoon", the rest answered "half past one". To the same question, asked half an hour later, two answered "two o'clock in the afternoon", the rest – "half past three". Half an hour later, four said "three o'clock in the afternoon", the rest – "half past three". How many spies could be among the respondents if it is known that each spy told the truth exactly once?

2 Digits

1. How many two-digit numbers are there in which the tens digit differs from the ones digit by more than 2?

2. How many ways can the numbers 1, 2, 3, 4, 5, 6, 7, 8 be split into pairs so that the differences of the higher and lower numbers in all pairs are the same?

3. The decimal notation of the number 5876 uses four consecutive digits. How many such four-digit numbers are there?

4. How many ways can you balance the scales using weights in 1, 2, 3, 5, 8, 13? Each of them is exactly one copy. It is not necessary to use all of them, but at least one of the weights should be on each part of the scale. Methods that differ only be changing parts of the scale are considered the same.

5. Very clever Petya and Vasya write out four-digit numbers. Petya writes out numbers in which the *first* digit is equal to the sum of the other three, and Vasya writes out those in which the *last* digit is equal to the sum of the other three. Who will write out more numbers and by how many?

3 Combinatorics

1. From a liter of milk, we can make 150 ml of cream, and from a liter of cream, we can make 300 g of butter. How many kilograms of butter can be obtained from 100 liters of milk?

2. A student has several 15-cent and 20-cent coins totalling some amount of money. He spent a fifth of this amount for a movie ticket and paid with two coins. He spent half of the remaining money on lunch paying for it in three coins. How many coins did the student have initially? List all possibilities.

3. Lesha and Gosha are digging a flower bed in their garden. They begin digging from opposite ends of the bed, moving towards each other. Gosha digs twice as fast as Lesha, but after excavating each meter he takes a 20-minute break. Lesha digs without interruptions. Gosha finishes digging his half of the bed after 100 minutes, and Lesha finishes digging her half of the bed after 2 hours. How long is the bed?

4. Once Nikolai noticed that his sister was exactly half his age. Now he is 3 times older than his sister was back then. In 25 years, Nikolai's combined age with his sister will be 85. How old is Nikolai now?

5. A teacher and several students are in a classroom. Find the number of students if you know the teacher is 24 years older than the average age of the students, and 20 years older than the average age of everyone in the classroom.

4 Number theory

1. In the sum of $1.1 + 1.2 + 1.3 + 1.4 + 1.5 + 1.6 + 1.7 + 1.8 + 1.9$, the decimal point in one addend was erased. The result of the sum then became an integer. What was the addend? List all possibilities.

2. In the decimal representation of a certain number, the digit "8" in the thousands place was changed to "3", and the number "7" in the ones place was changed to "4". The new number (with the corrected digits) divided by 57 equals 241 with a remainder of 37. Find the original number.

3. A natural number is "wonderful" if, when it is increased by 2, the sum of the digits of the resulting number is two times less than the sum of the digits of the original number. Find all the "wonderful" two-digit numbers.

4. Find the smallest natural number ending in 34, divisible by 34, and with a sum of digits equal to 34.

5. Find any three natural numbers such that the product of two of them is equal to the third, while in their decimal representations only numbers from 1 to 9 are used and exactly once.

5 6-grade geometry

1. Cut a 3×9 rectangle into eight squares.

2. A rectangle with one side three times longer than the other was cut into identical squares. It turned out that the sum of their perimeters is 6 times the perimeter of the original rectangle. How many squares could there be?

3. Draw eight points and connect them with segments so that the segments do not intersect and each point would be the vertex of exactly four segments.

4. A rope was folded into thirds, then it was fold into thirds again. After that, an incision that didn't coincide with fold lines was made. The rope fell into pieces, two of which were 2 cm and 6 cm long. Find the initial length of the rope.

5. Cut the shape shown in the figure into 6 equal parts, making cuts along the sides of the cells.