Homework Review

$x = 0.8\overline{23}$

$10x = 8.\overline{23}$

$1000x = 823.\overline{23}$

$1000x - 10x = 990x$

$8.23\overline{23} - 8.\overline{23} = 815$

$990x = 815$

$\boxed{x = \frac{815}{990}}$

$0.\overline{14}$

$0.12\overline{32}$

$x = 0.\overline{99}$ *

$100x = 99.\overline{99}$

$100x - x = 99x$

$99.\overline{99} - 0.\overline{99} = 99$

$99x = 99 \quad x = \frac{99}{99} = 1$? ?

2. Is there a difference? $0.72\overline{24}$ vs. $0.72\overline{242}$

make into fractions

$0.72242424\ldots$ same! $0.722424242\ldots$

3. Is this stuff useful?

maybe...

Sequences

1. What is a rational number?

   a number you can write as a fraction

2. What are some examples of irrational numbers?

   π, $\sqrt{2}$, $\sqrt{3}$

3. Let's look at some sequences.

   $\langle 0, 2, 4, 6, 8, 10, \ldots \rangle$ (+2, +2, +2)

   $n =$ 0, 1, 2, 3, 4, 5, ...

   What's the pattern? +2 each time
   even numbers

explicit definition

$a_n = 2 \cdot n$

$a_{15} = 30$

$n = 100 \rightarrow 200$

recursive definition

$a_{n+1} = a_n + 2$

$10 + 2$

$n = 5$

$a_{n+1} = a_6 \rightarrow 12$

$a_8 \rightarrow 16$

4. Write the first 6 perfect squares.

   $\langle 1, 4, 9, 16, 25, 36 \ldots \rangle$

   2^2 3^2 4^2

   $n =$ 0 1 2 3 4 5

Should we write an explicit or recursive definition? Why?

What is the definition?

$a_n = (n+1)^2$ explicit

5. $a_n = \frac{n(n+1)}{2}$

Is this an explicit or recursive definition?

explicit

Write the first 6 terms of the sequence.

$\langle 0, 1, 3, 6, 10, 15 \ldots \rangle$

6. $a_0 = 1$

$a_1 = 3$

$a_{n+2} = 2(a_{n+1} + a_n)$ $n=0$

$a_2 = 2(a_1 + a_0)$

$a_2 = 2(3+1)$

Is this an explicit or recursive definition?

recursive

Write the first 6 terms of the sequence.

$a_0 \quad a_1 \quad a_2 \quad a_3 \quad a_4 \quad a_5$

$\langle 1, 3, 8, 22, 60, 164 \ldots \rangle$

$n=1$

$a_{n+2} = 2(a_{n+1} + a_n)$

$a_3 = 2(a_2 + a_1)$

$a_3 = 2(8+3) = 22$

7. $\langle 5, 15, 45 \ldots \rangle$

$\times 3$ $\times 3$

$n =$ 0 1 2

what's the pattern?

$\times 3$

Should we write an explicit or recursive definition?

* or maybe both ;)

recursive definition:

$$a_0 = 5$$
$$a_{n+1} = a_n \cdot 3$$

explicit definition:

$$\boxed{a_n = 5 \cdot 3^n}$$

Sums of sequences

8. Arithmetic sequences... What are they?

$\langle 1, 4, 7, 10 \ldots \rangle$ +3 each time | $a_0 = 1$, $k = 3$

$\langle 0, 5, 10, 15, 20 \ldots \rangle$ +5 each time | $a_0 = 0$, $k = 5$

$\langle 1, 2, 3, 4 \ldots \rangle$ +1 each time | $a_0 = 1$, $k = 1$

9. $\langle 1, 4, 7, 10 \ldots \rangle$

What's the recursive definition?

$a_0 = 1$

$a_{n+1} = a_n + 3$

$\langle 1, 4, 7, 10, 13, \ldots \rangle$ $a_0 = 1$, $k = 3$

10. Find the sum, S, of the first n sequences in terms of a_0 and k $S_n = a_0 + a_1 + a_2 + \ldots + a_{n-1}$

$n = 2 \rightarrow S_2 = a_0 + a_1 = a_0 + (a_0 + 3) = a_0 + (a_0 + k)$

$S_2 = 2a_0 + k$

$n = 3 \rightarrow S_3 = a_0 + a_1 + a_2 = a_0 + (a_0 + k) + (a_0 + 2k)$

$S_3 = 3a_0 + 3k$

$n = 4 \rightarrow S_4 = a_0 + a_1 + a_2 + a_3 = a_0 + (a_0 + k) + (a_0 + 2k) + (a_0 + 3k)$

$S_4 = 4a_0 + 6k$

$n = 5 \rightarrow S_5 = a_0 + a_1 + a_2 + a_3 + a_4 = a_0 + (a_0 + k) + (a_0 + 2k) + (a_0 + 3k) + (a_0 + 4k)$

$S_5 = 5a_0 + 10k$

11. How can we generalize S_n for an arithmetic series?