Lesson 2: Weighings, Logic and Geometric Constructions

Konstantin Miagkov

In the following problems, "construct" means "construct using straightedge and compass".

Problem 1.
a) Given a segment AB on the plane, construct a point C such that ABC is an equilateral triangle using the ruler and compass.

b) Construct the midpoint of the segment AB.

Problem 2.
Given a point A and two rays out of it forming an angle α, construct the angle bisector of α.

Problem 3.
a) Given a line ℓ and a point P on this line, show how to construct a line through P perpendicular to ℓ.

b) Do the same if the point P is not on ℓ.

Problem 4.
Given a line ℓ and a point P not on ℓ, show how to construct a line through P parallel to ℓ.

Problem 5.
Suppose you have five positive integers, and you computed all ten of their pairwise sums. Is it possible that the ten pairwise sums all have different last digits?

Problem 6.
There are six coins on the table, one of them is fake and weighs differently than the real ones. Show how to determine the fake coin using a scale at most 3 times. In this problem, the scale simply shows the weight of all coins on it in ounces.

Problem 7.
A square is split into 100 rectangles using 9 vertical and 9 horizontal lines. Exactly 9 of those rectangles are squares – show that two of those squares have the same side length.