Fall Quarter Game

Nikita

1 Basic geometry

1. A cube frame was made from a 60 cm piece of wire. How much paper does it take to wrap it?

2. There are 4 pillars in the corners of the square pool. How can the pool be expanded so that the pillars remain on land, the area of the pool doubles, and the shape remains square?

3. How to fold a 2×2 square sheet of paper so that it can be cut into 4 1×1 squares in one stroke of scissors?

4. All the sides and bottom of the cardboard box (without the lid) are squares, each of which has side 1. Cut the box into three pieces so that they can be folded into a square.

5. It is known that there is a 10-gon, all sides of which lie on n lines. Find the smallest possible n (without a proof) and give an example of such a decagon.

2 Digits

1. A two-digit number is 5 times the sum of its digits. What is this number?

2. John has eight cards with numbers 1, 2, 3 and 4 - two with each number. He wants to construct a number out of these cards so that there is one digit between two ones, two digits between twos, three between threes, and four between fours. What are all the numbers that John could get?

3. Jack conceived three different digits from 1 to 9. Jim wrote down all possible two-digit numbers, in the decimal notation of which only these numbers were used. The sum of the written numbers is 231. Find the numbers conceived by Jack.

4. Think of a ten-digit number, the first digit of which would be equal to the number of ones in that number, the second digit is equal to the number of twos, the third digit is the number of triples, and so on. The ninth digit must be equal to the number of nines, and the tenth to the number of zeros.

5. There are two two-digit numbers that can be written one after the other to form a four-digit number that is divisible by the product of the two-digit numbers. What can these numbers be equal to? *(One example is enough)*

3   Text problems

1. Tilly, Willie, and Dilly were in the track and field race. At some point in time, it turned out that they were running next to each other, half of the participants in the race were running in front of them. and behind them - a third of the participants in the race. How many sportsmen were in the race?

2. Robert's current age is four times that of his sister's age when his age was twice of hers. In 8 years, the sum of their age will be 100. How old is Robert now?

3. Football matches Westwood Bears – Houston Cowboys, Houston Cowboys – Boston Beavers, Boston Beavers – Westwood Bears turned out to be very effective. Westwood Bears scored 60 goals. In total, Houston Cowboys conceded 80, and Boston Beavers scored as many goals as they conceded. What is the lowest number of goals scored in a Houston Cowboys – Boston Beavers match?

4. Abby and Amy walk along the downward-moving escalator, not missing the steps. Amy has time to take three steps, while Abby takes two. Amy, while descending, managed to take 45 steps, and Abby, while descending, managed to take only 40 steps. How many steps are there in the visible part of the escalator?

5. The cargo was first loaded into wagons with a capacity of 80 tons, but one wagon was not fully loaded.

   Then the cargo was reloaded into wagons with a capacity of 60 tons, but it took 8 more wagons and still one wagon was not fully loaded.

   Finally, the cargo was transferred to wagons with a capacity of 50 tons, it took another 5 more, but they were fully loaded.

   How many tons of cargo were there?

4 Verbal arithmetic

Cryptarithmetic is a type of mathematical game consisting of a mathematical equation among unknown numbers, whose digits are represented by letters. The goal is to identify the value of each letter.

Example:

$$
\begin{array}{ccccc}
  & S & E & N & D \\
+ & M & O & R & E \\
\hline
M & O & N & E & Y
\end{array}
$$

The unique solution to this puzzle is O = 0, M = 1, Y = 2, E = 5, N = 6, D = 7, R = 8, and S = 9.

$$
\begin{array}{ccccc}
  & 9 & 5 & 6 & 7 \\
+ & 1 & 0 & 8 & 5 \\
\hline
1 & 0 & 6 & 5 & 2
\end{array}
$$

Each letter should represent a different digit, and (as an ordinary arithmetic notation) the leading digit of a multi-digit number must not be zero. We don't guarantee that each cryptarithmetic puzzle here has a unique solution

1. Solve the puzzle

$$
\begin{array}{ccc}
  & G & O \\
+ & T & O \\
\hline
O & U & T
\end{array}
$$

2. Replace each letter with a digit so that
$$
\frac{O \times R \times M \times C}{G \times A \times M \times E}
$$
is an integer. What is the minimal integer value it can take?

3. How many solutions does the following puzzle have?

$$
\begin{array}{cccc}
T & H & I & S \\
+ & & I & S \\
\hline
E & A & S & Y
\end{array}
$$

4. Find at least one solution to
$$SIX + SIX + SIX = NINE + NINE.$$

5. Find at least one solution to
$$COVID - 2019 = MASKS.$$

5 Combinatorics

1. There are 17 latitudes (horizontal) and 36 longitudes (vertical) on the globe. How many sections do these lines divide the surface of the globe into?

2. Twelve empty chairs are in a row. Whenever a person sits down in an empty chair, exactly one of their neighbors gets up and leaves (unless both adjacent chairs are empty). What is the maximum number of people who can be seated at the same time if all the chairs are initially empty?

3. In a 4×4 grid, the vertices of all cells are marked, creating a new grid of 25 marked points. How many triplets can you choose of 3 colinear (lying on the same line) points?

4. 16 people are participating in a tug-of-war competition. Every day the organizers of the tournament divide the participants into two equally sized teams. What is the minimum number of days the organizers need to ensure that every pair of participants is on opposing teams at least once?

5. 12 people are taking part in a chess tournament. Each participant plays every other participant exactly once. Contestants earn 1 point for each victory, 0.5 points for each draw, and 0 points for each defeat. To earn the title "Master of Chess", a contestant must score more than 70% of the theoretical maximum number of points (had they won all games). What is the largest number of participants who can receive the "Master of Chess" title?