FROM DECIMALS TO FRACTIONS

LAMC BEGINNERS GROUP - 10/02/11

(1) The denominators of fractions in Problem 7 from last time all had a certain property called **Property X**, which the denominators in Problem 8 didn't have. Here are some more numbers with Property X:

- 20, 5, 4, 1000, 16, 50, 2, 800, 125, 10

The following numbers do **NOT** have Property X:

- 30, 3, 9, 1001, 36, 55, 7, 700, 130, 15

   (a) Factor into primes, each of the numbers listed above with Property X:

   (b) Factor into primes, each of the numbers listed above that did **not** have Property X (Hint: 1001 is divisible by 13):

(2) Looking back at Problems 7 and 8 from last time, what does this tell you about decimal representations and Property X? Fill in the blanks:

- The decimal representation of a fraction whose denominator HAS Property X is

  _____.

- The decimal representation of a fraction whose denominator DOESN'T have Property X is

  _____.

(3) Convert the following decimals into fractions. Make sure to put them in lowest terms by eliminating any common factors of the numerator and denominator.
(a) .89

(b) .35

(c) .0125

(d) 44.44

(4) Convert the following fractions into decimals–do you see a pattern? Complete the pattern with the next few fractions and their decimal representations.
(a) 1/9

(b) 2/9

(c) 3/9

(d) 4/9

(e) 5/9

(f)

(g)

(h)

(i)

(5) Building on the pattern you saw in the previous problem, see if you can convert these infinite decimals into fractions (in lowest terms):

(a) 3.33333333...

(b) 17.55555555...

(c) 2.0088888888...

(6) Albert is thinking of a secret number (not necessarily a whole number!). If you multiply Albert's number by 10, you get the same result as you would if you added 6 to it. What is Albert's number? Give your answer as both a fraction in lowest terms, and a decimal.

(7) Dustin is also thinking of a number. If you multiply Dustin's number by 100, you get the same result as you would if you added 100 to Dustin's number. What is Dustin's number? Give your answer as both a fraction in lowest terms, and a decimal.

(8) Maria is thinking of a number as well. If you multiply Maria's number by 1000, you get the same result as if you added 37 to it. What is Maria's number? Give your answer as both a fraction in lowest terms and a decimal.

(9) Sony is thinking of three whole numbers between 1 and 10, inclusive. He tells Clint their sum and product. Clint thinks for a while and then tells Sony, "Dude, that's not enough information for me to figure out your numbers." Then Sony then tells Clint that among the three numbers, two of them are consecutive (that is, right next to each other). Clint says, "Aha, your numbers are X, Y, and Z!" (except Clint says the actual numbers instead of "X," "Y," and "Z.") What were Sony's numbers?

(10) Consider the decimal number x = 0.22222222..., which has 1 repeating digit.
  (a) What is 10x as a decimal?

  (b) What is 10x - x as a decimal?

  (c) Since 10x - x = 9x, what is the value of 9x?

  (d) Based on part (c), what is the representation of x as a fraction?

(11) Try using the process from Problem 10 to write x = 0.66666666... as a fraction. (Write down the steps of the process even if you already know the answer!)

(12) Consider the number y = 0.36363636..., which has 2 repeating digits.
  (a) What is 100y as a decimal?

  (b) What is 100y - y as a decimal?

  (c) Since 100y - y = 99y, what is the value of 99y?

  (d) Based on part (c), what is the representation of y as a fraction?

(13) Try using the process from Problem 12 to write y = 0.05050505... as a fraction.

(14) Consider the decimal z = 0.148148148148..., which has 3 repeating digits. Use a similar process to that in Problems 10 and 12, to write z as a fraction. (Hint: Start by multiplying z by 1000, since it has 3 repeating digits.)

(15) The decimal 0.abcabcabcabc... has 3 unknown repeating digits. How could you write it as a fraction? (Your fraction will have to use a, b, and c to represent the unknown digits also! It does not have to be in lowest terms.)

(16) Now let's try a decimal like w = 0.466666666..., which starts with a 4 and then repeats the digit 6.

(a) What is 10w - 4 as a decimal?

(b) What is 10w - 4 as a fraction?

(c) Use part (b) to help you write w itself as a fraction.

(17) If you know a little algebra, try to find a simple form for the value of

$$x = \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \ldots}}}}$$

(Hint: First square both sides...)