

HOTEL INFINITY I

JUNIOR CIRCLE 09/25/2011

Imagine you are the owner of Hotel *Infinity*. This hotel is a very interesting place, it has the following properties:

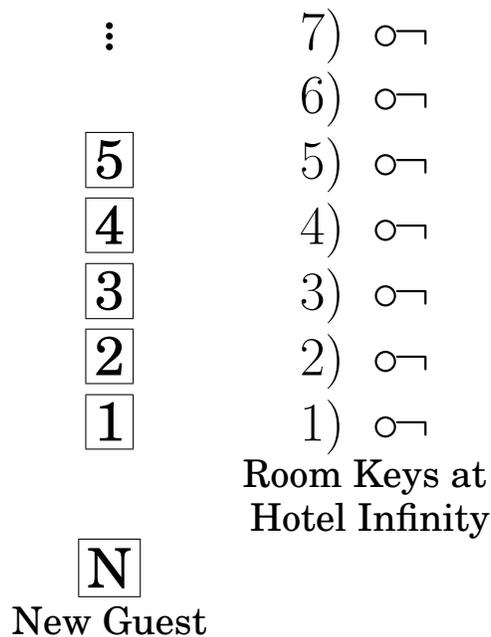
- There are infinitely many rooms in the hotel (numbered 1, 2, 3, 4, . . .);
- Every room is on a different floor;
- Only one person can stay in each room at any moment;
- The guests are required to move to another room if asked to do so;

Here are some problems for you to solve:

- (1) In the morning, all the rooms in the hotel are taken. Then one more guest arrives. Can you accommodate the guest?

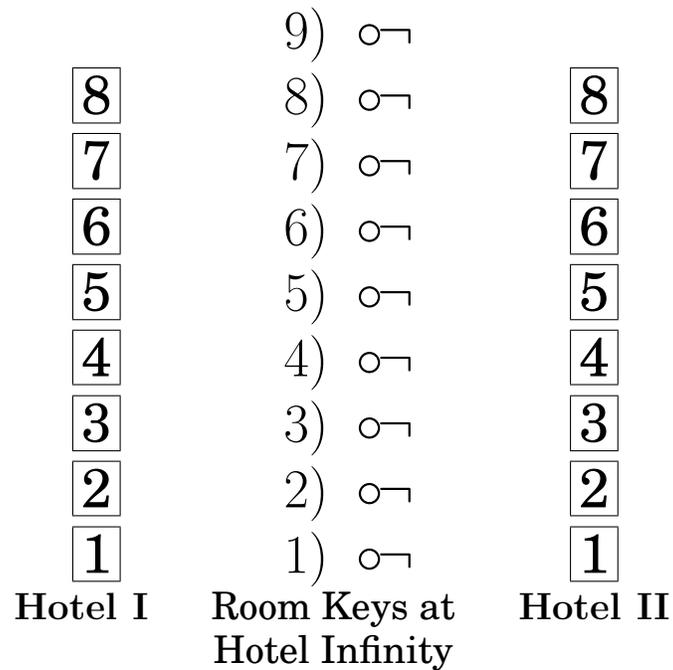
You allocate rooms by distributing keys.

Use the sketch below to display your solution to this problem.



-
- (2) After this, things are quiet for a while. Then, 5 guests arrive. Can you find room for each of the new guests as well? Draw a picture and explain how to do it.

(3) Now imagine you have two siblings, each owning a hotel almost exactly the same as yours (with infinitely many rooms). However, they are located in different places. Both of your siblings' hotels are filled, while yours is empty. All the guests from *both* of these other infinite hotels want to relocate to yours to be closer to the beach. Can you accommodate the guests from both of these hotels into your one Hotel Infinity? If so, distribute keys to the guests.



-
- (4) Your friend in the nearest galaxy has his own hotel *Double Infinity*. In his hotel, there are also infinitely many floors numbered $1, 2, 3, \dots$. However, each floor has 2 rooms, so that
- rooms 1 and 2 are on floor 1;
 - rooms 3 and 4 are on floor 2;
 - rooms 5 and 6 are on floor 3;
- (a) What rooms are on the 100th floor?
- (b) Which floor has rooms 2011 and 2012 on it?
- (c) What rooms are on the n th floor?
- (d) Your friend says that his hotel has twice as many rooms as yours. Do you think he is right? Explain why or why not.

- (e) One day, his hotel is completely full. Since your hotel is empty, you are offering rooms at a discounted price. All the guests from *Double Infinity* immediately decided to move to your hotel. Can you accommodate all of them? How? You may use the post-its and index cards to help you model this problem.

(5) Can you find place for all the guests from 3 infinite hotels in your Hotel Infinity? (Your hotel is empty at the beginning). What about 4 infinite hotels? What about 5?

(6) Your Hotel Infinity is full. However, you have promised to accommodate all guests from your brother's infinite hotel. How can you do it? Remember, your guests will move to a different room right away at your request.