17 Möbius strip

Materials for the lesson and homework: a couple of regular pencils, a red and blue pencils, an eraser, a pencil sharpener, craft sticks, paper, scissors, paper glue or sticky tape.
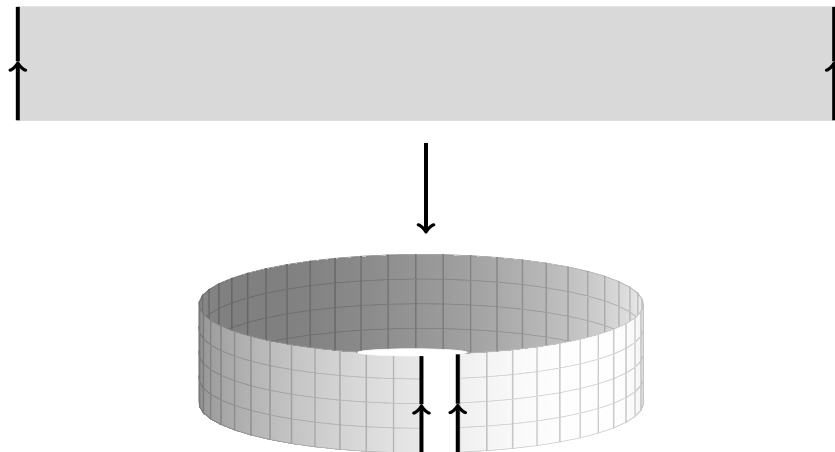
Warm-up

**Problem 17.1** *The number sentence below is made of craft sticks:*

$$VII + III = V$$

Move one stick to correct it.

17.1 Lesson

A standard US letter sheet measures 8.5×11 inches. Please cut two 1×11 inches strips of paper out of it. Glue the first strip into a cylinder as shown on the picture below.

Give the second strip a minimal twist needed to align the arrows on the shorter sides, then glue as shown on the picture below. The resulting surface is called a *Möbius strip* after its inventor, a German astronomer and mathematician August Ferdinand Möbius (1790-1868).
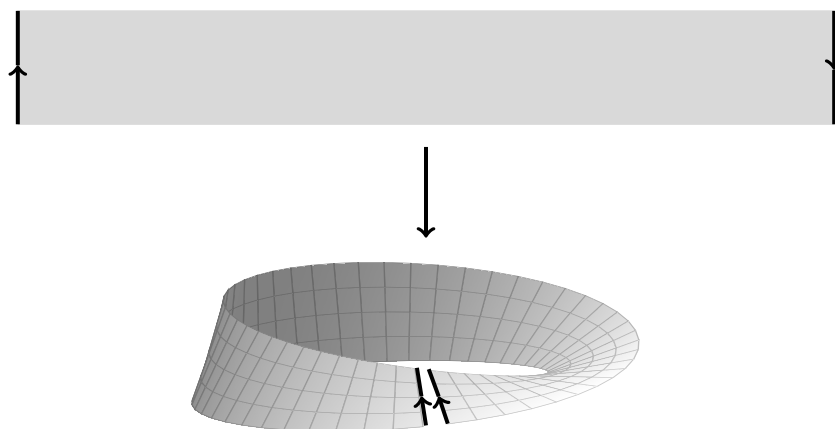
The goal of this lesson is to study the Möbius strip by comparing it to the cylinder.

Problem 17.2

• *Draw the points H and S on your cylinder as shown on the picture below. The point H represents Home where a little bug lives. The point S represents School. The bug cannot crawl over the edge.*
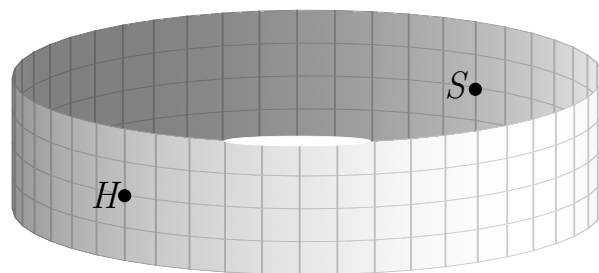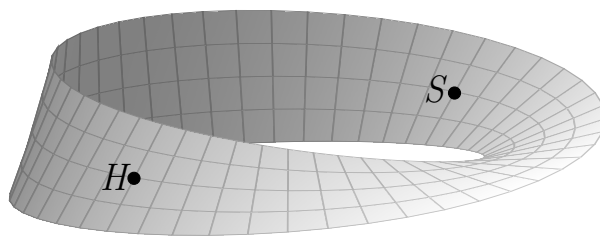
Can the bug get from Home to School? Please circle the correct answer.

Yes   *No*

- *If you think the bug can get from Home to School, please draw a path connecting the point S to the point H on your cylinder.*

- *Draw the points H and S on your Möbius strip as shown on the picture below. Once again, the point H represents Home where a little bug lives. The point S represents School. The bug cannot crawl over the edge.*

Can the bug get from Home to School? Please circle the correct answer.

Yes   *No*

- *If you think the bug can get from Home to School, please draw a path connecting the point S to the point H on your Möbius strip.*

The following two problems explore and compare midlines on a cylinder and Möbius strip.

Problem 17.3

• *A bug crawls on the cylinder so that its distance to the edges is the same at all times. Draw a path of the bug on your cylinder.*

• *What shape is the path?*

• *Recall that the length of the original strip was 11 inches. Assume you wasted no length to glue narrow sides of the strip together. What is the length of the midline?*

$$\textit{The midline length is \underline{\qquad} inches.}$$

• *Another bug wants to crawl on the cylinder so that its distance to the edges is the same at all times. Can the second bug find a path that does not meet the path of the first bug? Please circle the correct answer.*

$$\textit{Yes} \qquad\qquad \textit{No}$$

• *If you circled Yes, draw the path on your cylinder. If you circled No, please explain why.*

Problem 17.4

• *A bug crawls on the Möbius strip so that its distance to the edges is the same at all times. Draw a path of the bug on your Möbius strip.*

• *What shape is the path?*

• *Recall that the length of the original strip was 11 inches. Assume you wasted no length to glue narrow sides of the strip together. What is the*

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length of the midline?

The midline length is _____ inches.

• *Another bug wants to crawl on the Möbius strip so that its distance to the edges is the same at all times. Can the second bug find a path that does not meet the path of the first bug? Please circle the correct answer.*

Yes   No

• *If you circled Yes, draw the path on your cylinder. If you circled No, please explain why.*

Problem 17.5 *Use your index finger to trace the boundary lines on a cylinder and Möbius strip. Then answer the following questions.*

• *How many boundary lines does a cylinder have?*

A cylinder has _____ boundary lines.

• *How many boundary lines does a Möbius strip have?*

A Möbius strip has _____ boundary line.

Problem 17.6

• *Imagine that you cut a cylinder with scissors along the midline. What do you think you will you get?*

• *Cut a cylinder with scissors along the midline. What do you get?*

- *Imagine that you cut a Möbius strip with scissors along the midline. What do you think you will you get?*

- *Cut a Möbius strip with scissors along the midline. What do you get?*

To make one more model of a cylinder and Möbius strip, cut two more 11×1 inches strips. Color one side of each of the strips then glue them into the corresponding shapes as shown on pages 1 and 2.

Problem 17.7

- *Make sure one side of the cylinder is colored so that there are no white spots left. Can you color the other side of the cylinder using a different color? Please circle the correct answer.*

$$Yes \qquad\qquad No$$

- *Make sure one side of the Möbius strip is colored so that there are no white spots left. Is there left any place for a different color anywhere on the Möbius strip? Please circle the correct answer.*

$$Yes \qquad\qquad No$$

- *How many sides does a cylinder have?*

A cylinder has _____ sides.

- *How many sides does a Möbius strip have?*

A Möbius strip has _____ side.

Problem 17.8 ❭ *Can you cut a square into six smaller squares not necessarily having the same size? If you think you can, please draw a picture in the space below. If you think that it is not possible, please explain why.*

17.2 Homework

Finish all the problems from class. Explain to your parents how to make a Möbius strip and why it has only one side.

Problem 17.9 *Alice and Bob ate all the chocolates from a square box. Bob ate all the twelve chocolates along the rim. There was one layer of chocolates in the box. All the chocolates had the same shape and size. How many chocolates did Alice eat?*