Lesson 4: Invariants and Geometric Constructions

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1 From Last Week

Problem 2.
All four corners of an 8×8 board are colored black, and the rest of the squares are white. You are allowed to change the colors of all squares in a single row or column. Show that it is impossible to use such operations to make all the squares white.

Problem 3.
Consider the numbers 1 through 6 placed on a circle in order. It is allowed to add one to three consecutive numbers, or to subtract one from three numbers no two of which are adjacent. Is it possible to use these operations to make all six numbers equal?

Problem 5.
Let P be a point outside of a given circle. Show how to construct the two tangent lines from P to the circle.

2 New Problems

Problem 1 (Use this problem as a hint for L3.2)**.**
Consider a 3×3 square with the lower-left corner colored black, and the rest colored white. Is it possible to make all squares white by repeatedly changing all the colors in a row or column?

Problem 2.
In the alphabet of the "Mumbo-Yumbo" tribe there are only two letters – M and U. Two words have the same meaning if and only if one of them can be transformed into the other by inserting or removing the strings "MU" and "UUMM" at arbitrary places in the word arbitrarily many times. Do the words "UMM" and "MUU" have the same meaning?

Problem 3.
Given two segments AB and CD, construct a right triangle with hypothenuse equal to AB and one of the legs equal to CD.
Hint for both this and L3.5: for a given segment MN, the points P such that $\angle MPN = 90°$ are precisely the points lying on the circle with diameter MN.