Homework 2: Weighings, Logic and Geometric Constructions

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1   Homework

Problem 1.
The are 12 knights and knaves in the room. The first person said: "There are no knights in the room." The second person said "There is at most one knight in the room." And so on, until the 12th person said "There are at most eleven knights in the room." How many knights were in the room?

Problem 2.
Given a point A outside of the line ℓ and a point B on ℓ, construct a circle which goes through A and is tangent to ℓ at B.

2   Reading

Solution 1 (H1.1)**.**
We definitely know that the last coin is fake since there is at least one fake coin and all real coins are to the right of all fake ones. Similarly, the first coins has to be real. Now let us compare coins $1, 4$ with coins $2, 3$ using the balance scale. If $2, 3$ are lighter, they both have to be fake. If they are heavier, they must both be real. If the weights are equal, one of $2, 3$ is real and one is fake. But then 2 is real and 3 is fake, and we are done.

Solution 2 (L1.4)**.**
We must have exactly 15 friendship pairs, each consisting of one knight and one knave. If two such friends are sitting next to each other, the knight says "Yes" and the knave says "No". If the are not sitting next to each other, the knight says "No" and the knave says "Yes". Either way, each pair produces one "Yes" and one "No". This means that there will be a total of 15 of each answer, and the answer to the problem is zero.