# Grid Luck

For the following problems, use grid paper to help figure out what's going on and get to the answers. Most problems are based on original "Grid Luck" problem set by Tatiana Shubin (shubin@math.sjsu.edu).

1. Tetris pieces are made of all the different ways you can connect 4 squares together. (Rotating doesn't make a difference, but two pieces which are non-identical mirror images are considered different.) Can you remember/figure out all the Tetris pieces?

2. On the grid paper, draw a rectangle and one of its diagonals. How many grid squares are crossed by the diagonal?

3. (a) How many *squares* are there in a $2 \times 3$ rectangle? (Hint: It's more than six!) How many squares in a $5 \times 8$ rectangle?

   (b) How many *rectangles* are there in a $2 \times 3$ rectangle? In a $5 \times 8$ rectangle?

4. Given two equal-sized squares, can you cut them into a number of pieces and rearrange them so that they form one larger square? What is the smallest number of pieces you need?

5. Find an infinitely long path in the plane having the property that starting from *any* point in the plane one can reach the path by moving a total distance of less than 1 unit.

6. Is it possible to dissect a square into 2 squares? 4 squares? 6 squares? 7 squares? Can you decide for which numbers $n$ it is possible to dissect a square into $n$ smaller squares?

7. What if Tetris pieces were made of all the different ways you could connect 5 squares together (Same deal as before: rotating doesn't make new pieces, but reflecting can.) How many of these can you draw?

8. There are 2 ways to place 2 dots into an $2 \times 2$ grid so that each dot has its own row and column. How many ways are there to place 3 dots in a $3 \times 3$ grid so that no two dots are in the same row or column? What about 4 dots in a $4 \times 4$ grid? Can you find a pattern? What about $n$ dots in an $n \times n$ grid?

9. Draw a rectangle and trace the path of a billiard ball that begins in the lower left corner and initially travels upward at a 45-degree angle (diagonally). If the ball bounces off each wall at a perfect 45-degree angle, which corner does the ball reach first? What fraction of all unit squares does the ball pass through on the way? Start your experiment with a $3 \times 5$ rectangle, then choose other dimensions.

10. (a) How many points in the plane can you find such that the distance between any two of them is a whole number?

    (b) How many points can you find, *no three of which lie on a single line*, such that the distance between any two of them is a whole number?